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PERSPECTIVES
ON LANGUAGE AND LITERACY

A Quarterly Publication of The International Dyslexia Association

Volume 37, No. 2

MATHEMATICAL DIFFICULTIES IN SCHOOL AGE CHILDREN

- Number Concepts and Combinations
- Working Memory
- Procedural Skills
- Conceptual Mastery
- Meaningful Assessment
- Motivation and Engagement
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PERSPECTIVES

on Language and Literacy

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In a way that creates hope, possibility, and partnership…

So that every individual has the opportunity to lead a productive and fulfilling life, and society benefits from the resource that is liberated.

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IDA's membership is comprised of people with dyslexia and their families, educators, diagnosticians, physicians, and other professionals in the field. The headquarters office in Baltimore, Maryland is a clearinghouse of valuable information and provides information and referral services to thousands of people every year. IDA's Annual Conference attracts thousands of outstanding researchers, clinicians, parents, teachers, psychologists, educational therapists, and people with dyslexia.

Mathematical Difficulties in School Age Children

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Letter from the President

Dear Colleagues and Friends,

Reading and arithmetic are cultural inventions that have come to us relatively recently. When considered in the context of some of the other abilities that we have from birth (visual and auditory discrimination) or acquire in our repertoire of skills during development (spoken language), reading and arithmetic are not attained easily; we have to make a concerted effort to learn them once other contributing cognitive and sensorimotor skills have developed sufficiently to support them.

You have probably heard people say that there is nothing natural about teaching a child to learn to read. There are no brain regions that are innately programmed to read (unlike the many regions of the brain that are set up to deal with specific tasks from birth). The reason we learn to read is because our society places a great deal of importance on it and our environment requires us to do it on a regular basis. Reading has to be taught and, as has been demonstrated through functional brain imaging research, during a protracted period of explicit instructions, the brain regions that are engaged during word reading change as the reader becomes more proficient. Further, the brain regions that make up the "reading network" in a skilled reader are areas that were not designed to read, but were intended for other kinds of functions.

The reason they became reoriented to the reading process has something to do with the properties they have that make them amenable to be drawn into reading (perhaps their involvement in language processing or visual analysis of objects). How this occurs exactly is not known. While it is often stated that our brains are "learning machines" that can learn the many skills demanded by our cultural situation, it has also been suggested that brain areas that play a role in reading do so not because of their capacity to support learning *per se*. Instead, these regions subserve functions that are sufficiently close to reading and become "recycled" into performing this task (see the work of Stanislas Dehaene for a very fascinating account of the recycling theory).

The same concept would apply to arithmetic. Like reading, arithmetic is optional and it is achieved through learning. Yet there are certain skill sets that children bring to the table that provide an important basis on which to build arithmetic concepts. That is, "number sense" is to arithmetic what "phonological sensitivity" is to reading. Preschool aged children possess the capacity to readily manipulate nonsymbolic quantity, as evidenced by their ability to discriminate magnitude and even perform arithmetic operations on dot arrays and sequences of sounds. However, as children begin to learn the symbolic representation of numbers (our cultural representation of quantity) they take on a uniquely human task. As for reading, areas in the brain that are responsible for non-symbolic and symbolic arithmetic processes have been identified and studied over development. Age-related increases of activity have been observed during a symbolic magnitude judgment task, suggesting a developmental specialization of the left hemisphere for the representation of symbolic magnitude, while no such differences are observed when comparing adults and children during non-symbolic magnitude judgment. When comparing children and adults on arithmetic tasks, the mature brain relies on frontal brain regions to perform simple subtractions and additions, while children use regions in the back of their brain. Curiously, this developmental trajectory is somewhat different from reading, where frontal brain areas are engaged in the older, more proficient reader.

How does this complex mixture of biological predisposition and cultural context play out in the reality of the classroom? Our understanding of why children are successful in these skills has been greatly increased by research efforts over the last 40 years for reading, and more recently for mathematics. Specifically, there has been a deliberate effort to fund more research in the area of mathematics, recognizing that the need to have better ways of predicting, identifying, and instructing children with difficulties in this domain. This issue of *Perspectives on Language and Literacy*, spearheaded by Michèle Mazzocco, provides a very timely update of these research efforts. It has been of great interest to researchers and practitioners to have better tools to understand the development of mathematics and to uncover the reasons why some children stumble when it comes to learning arithmetic. Also, why is it that some children are challenged by both reading and math? Is it because of something that both of these domains share and that is weaker in these students (such as working memory)? What interventions and exercises promote numeracy and mathematics outcome? Many of the answers to these questions are in this issue, and I thank Dr. Mazzocco and her colleagues for bringing this complex body of work into a succinct summary for our readers.

Sincerely yours,

Guinevere Eden

Guinevere Eden, D.Phil., President

A Gift in Memory of K. Bryant Wicke, Former Chairman of the Tremaine Foundation

The International Dyslexia Association (IDA) recently received a significant gift to improve the quality and efficiency of its website. This gift enables IDA to utilize its *Knowledge and Practices Standards for Teachers of Reading* to identify, evaluate, and endorse qualified tutoring and testing providers by funding an upgraded, comprehensive database of certified professionals in a more sophisticated format. This upgrade will provide 24/7 access to a list of qualified professionals within a certain radius simply by inputting a zip code.

Born in 1961, K. Bryant Wicke was an avid sportsman and philanthropist, serving on the Tremaine Foundation board for over 20 years. He suddenly and tragically died in 2009 from injuries resulting from an automobile accident.

The motivation behind the gift to IDA is to assist parents and families of children confronting learning difficulties. It recognizes IDA's outstanding contributions to the field of learning disabilities by providing additional resources to continue its longstanding mission to improve the lives of individuals, both young and old, with dyslexia. But the heartfelt element of this gift is in honor of a wonderful man who loved life, his family, friends, and those young people who deserve a hand in their struggle with learning disabilities.

As a member of the Tremaine family, Bryant was a part of a substantive tradition of philanthropy directed at dyslexia and learning disabilities. For many years, the Tremaine Foundation has provided the field with vision, thinking, and influence that have been the benchmark for others. The Foundation's openness and forthright attitude about the challenges facing our community reflect the personalities on its board. Bryant was very proud to serve with his family members and to have the opportunity to chair the Foundation's board.

Bryant brought to the table a level of passion and spirit that comes only through the adversity and pain one feels when dealing with dyslexia firsthand. Bryant's attitude toward life showed a drive to succeed and to persuade and encourage others with learning disabilities that they were indeed remarkable individuals, with skills and insights that would see them clear of any hurdle before them. His long trusteeship at the Hillside School, a middle school for boys, was another example of his commitment to reach out to his peers. He was also a generous donor to Linden Hill, a small and nurturing boarding school for boys with learning disabilities.

Young people need spiritual mentors like Bryant Wicke to sustain them when they can only see their own faults and weaknesses. Through this gift and the memories of everyone who knew him, Bryant is still with us, supporting all those who work in the field of learning disabilities and reminding us, repeatedly, that we can if we will.

— Henry Sinclair Sherrill

Theme Editor's Summary

Mathematical Difficulties in School Age Children

by Michèle M. M. Mazzocco

Researchers of children's mathematical difficulties (MD) are frequently asked the very questions they seek to answer:

- Why do children struggle with mathematics?
- What is the best way to teach children who struggle with mathematics?

In this issue of *Perspectives on Language and Literacy*, we summarize some of the current knowledge regarding *why* children struggle with mathematics and *what* can be done to foster children's mathematical success.

The short answer to these questions is simply, "it depends." It depends on characteristics of students and teachers, how and what students are learning, and how and what teachers are teaching. Fortunately, there is some evidence-based knowledge to elaborate on these general responses, despite the fact that many factors and their influences are unknown or untested. In this issue we address students' specific and general cognitive skills and their motivation to learn mathematics, but we also emphasize the skills and knowledge of teachers. We consider how these factors may promote or hinder how readily students learn, remember, and execute mathematics and how well they transfer skills and knowledge across components of mathematics knowledge. Although several influences on mathematics learning have been identified, the mechanisms and causal pathways associated with these factors are not yet fully understood.

Our responses to the two questions posed above are sometimes partial, or even tentative, in view of the paucity of research on MD (e.g., Gersten, Clark, & Mazzocco, 2007). Still, it is apparent that a complete response to each question is multifaceted, as is illustrated by the range of topics presented across the contributions to this special issue.

Why do children struggle with mathematics and how do we teach these children? It depends on the aspect of mathematics being taught, learned, or performed. This issue begins with Drs. Powell, Fuchs, and Fuchs summarizing their intervention research on children's fluency with number combination skills (sometimes misleadingly referred to as "math facts"). They reveal that the effectiveness of attempts to tutor students who struggle with number combinations depends on the nature of instruction offered during number combinations tutoring, whether students are taught backup strategies for when number combinations are not readily retrieved as facts, and whether students have opportunities to practice number combination skills. It is possible that the relationships discovered by these researchers further depends on whether students struggling with mathematics are also struggling readers, as these authors discuss.

Next, Daniel Berch describes the role of children's working memory in learning, performing, and teaching mathematics. Describing working memory as "the mind's workspace," Dr. Berch articulates how mathematics learning and performance depend on individual and developmental differences in children's working memory capacities, on differences in the working memory demands across math tasks, and on how well teachers recognize and adapt to the differential abilities or demands related to working memory. Thus, Dr. Berch exemplifies that *why* children struggle with math depends on characteristics of children (e.g., their working memory skills) and on the instruction they receive, and *how* to teach struggling learners depends on characteristics of the learners themselves and the math problems they face (e.g., number combinations versus long division).

An explanation for why students struggle with math also depends on whether students have a conceptual understanding of the mathematics they are learning. In her article, Julie Booth emphasizes the importance of differentiating, attending to, and acquiring both procedural and conceptual understanding of mathematics. When learning (or instruction) focuses on procedures (e.g., what to do) but not concepts (e.g., why you do it), students may develop misconceptions about mathematics, which can further hinder their learning. Dr. Booth explains why "eliminating student misconceptions should be a critical goal for successful mathematics instruction," while also "cultivating strong conceptual understanding." She exemplifies routes towards achieving these goals, including the use of meaningful assessments of *why* students struggle with math and recognizing that what we learn from assessments depends on how we ask the questions. "One can acquire very different types of information about what students know [about a mathematics problem] by the way that one asks the students to think about the problem" (Booth, this issue).

Teachers' effectiveness in promoting conceptual learning depends on their own knowledge of mathematics content and of mathematics instruction, as described by Melissa Murphy and her colleagues in this issue. These authors stress that, "the magnitude of the effect of teacher knowledge on student [mathematics] achievement is comparable to the effect of… [students'] absence rate…and socioeconomic status…" Moreover, effects of teacher knowledge are compounded by other risk factors. For example, as Dr. Murphy and colleagues explain, "less mathematically knowledgeable teachers tend to be employed in school districts that serve low income communities," and it is well established that low socioeconomic status is a risk factor for poor mathematics achievement outcomes (Jordan & Levine, 2009).

Whether students struggle or sail through mathematics depends on these and other characteristics of students, teachers, and learning environments; but even when these factors

Continued on page 8

are present at optimal levels, learning may be compromised if students are not engaged. Laurie Hanich explains that cognitive and motivational behaviors are "dynamically related, [such that] children's belief systems in turn may affect adaptive behaviors for mathematics learning" (Hanich, this issue). Although this principle applies to learning across domains, Dr. Hanich explains that repeated failures with mathematics may further impede learning in students who struggle with mathematics by prompting them to avoid mathematical instructional opportunities and tasks (intentionally or not). Hanich explains that engagement is driven by child characteristics and the opportunities educators provide to foster engagement.

Finally, why students struggle with mathematics and how teachers should respond to them also depends on what we mean by "struggling with mathematics" and on the cognitive skills that underlie children's math learning and performance. Not all students who struggle with mathematics struggle in the same way or for the same reason, and not all children who struggle with mathematics have a specific mathematical learning *disability* (MLD) if we define MLD as a domain-specific deficit in understanding or processing numerical information. However, this term, which is often and accurately used synonymously with *developmental dyscalculia,* is not defined consistently in the literature (as reviewed elsewhere, for example, Mazzocco, 2007). Most children with mathematical *difficulties* (MD) do not have MLD, as supported by the lower prevalence rate for the latter. Approximately 10% of children have MLD (as reviewed by Shalev, 2007). In contrast, far higher rates of mathematical difficulty are implicated by large scale national reports, such as the *Nation's Report Card* that claims 27% of U.S. eighth graders fail to demonstrate mastery of basic mathematics, and 66% fail to achieve mathematical proficiency (e.g., NCES, 2009). It is unlikely that all of these students have MLD. Thus, although difficulties with mathematics may be secondary to either domain-general cognitive deficits (e.g., working memory) or environmentally influenced learning obstacles (e.g., the effects of poverty), they may also be linked to numerical deficits characteristic of MLD. Of course, such difficulties may also result from some combination of these separate factors.

Researchers have found important differences in the cognitive characteristics of children who meet criteria for MLD versus MD (e.g., Geary et al., 2007; Murphy et al., 2007), just as there are differences in children whose mathematical difficulties do, or do not, co-occur with reading disabilities (e.g., Jordan et al, 2003). Thus, in the final article of this special issue, I focus on sources of individual differences in number skills seen among and between groups of children with MD, including the subset of these children who have MLD.

To summarize, there are many reasons why students struggle with mathematics. We address several of them in this special issue. Some additional factors have been addressed elsewhere; for instance, the importance of language in mathematics classrooms was addressed in an earlier volume of *Perspectives* (i.e., Volume 34(2) by Woodward, Montague, Jitendra, & Baxter, 2008); while other factors remain to be discovered. Like the goal that underlies ongoing research on children's mathematical learning difficulties, the objective of this special issue is to support *your* ability to answer the very questions that guide our research, in reference to individual students in your charge. In other words, we hope that the articles that follow will help you determine possible sources of mathematics difficulty for individual students you encounter, and will guide your efforts to support each student's learning of mathematics skills, procedures, and concepts.

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Number Combinations Remediation for Students with Mathematics Difficulty

by Sarah R. Powell, Lynn S. Fuchs, and Doug Fuchs

Students with mathematics difficulty (MD) often struggle with developing fluency with number combinations. Problems with number combinations can lead to difficulties with computation, geometry, algebra, problem solving, and most other mathematics topics. In this article, we discuss fluency with number combinations and its importance for mathematics development. We also discuss why students struggle with learning number combinations. Then, we describe four studies that we conducted to remediate number combinations difficulties. Finally, we offer suggestions for helping students with MD learn number combinations.

Before proceeding, we comment on the use of the term MD. We, similar to other researchers, have defined low performance in mathematics as *mathematics difficulty*. In our studies, students with mathematics difficulty performed below the 26th percentile on a standardized test of mathematics. Some of these students with MD had an official school diagnosis of *mathematics learning disability*, but many of the students with MD struggled with low mathematics performance without an official diagnosis.

What Are Number Combinations?

Number combinations are sometimes referred to as basic facts or math facts. We use the term *number combinations* to show that students can work and solve these problems; that is, these problems do not have to be recalled as a fact from memory. Number combinations comprise 390 addition, subtraction, multiplication, and division number combinations that are some of the basic building blocks of mathematics (Hudson & Miller, 2006). See Table 1 for examples of each of the following types of number combinations. Addition number combinations have addends of 0–9 and a sum of 0–18. Subtraction number combinations have minuends of 0–18 and subtrahends and differences of 0–9. Multiplication number combinations have factors of 0–9 and products ranging from 0–81, whereas division number combinations have dividends of 0–81, divisors of 1–9, and quotients of 0–9. Generally, addition number combinations are more likely to be retrieved from memory (rather than calculated) compared with the three other types of number combinations. Because our work on number combinations has focused exclusively on addition and subtraction, we limit our discussion to addition and subtraction combinations in this article.

Why Are Number Combinations Important?

It is important that students know (i.e., retrieve from memory) or be able to solve (i.e., quickly calculate an answer) number combinations because number combinations are necessary for solving most other types of math problems: computation, money, measurement, geometry, algebra, and problem solving. If students do not know or cannot calculate the answer to a number combination with relative ease, then solving other math problems is more difficult or impossible (Kroesbergen & Van Luit, 2003). Think of how difficult it might be to subtract coins (e.g., 17 pennies minus 9 pennies) or calculate the perimeter of a rectangle (e.g., 8 inches plus 6 inches) if a student does not demonstrate fluency with number combinations. Additionally, students with weak number combinations skill may develop anxiety about mathematics (Hudson & Miller, 2006), which may promote negative attitudes about mathematics and avoidance of situations where mathematics is necessary (as discussed by Laurie Hanich, this issue).

Students Who Struggle with Number Combinations

Many students who struggle with mathematics demonstrate a lack of skill with number combinations (Andersson, 2008; Geary, Hamson, & Hoard, 2000; Hanich, Jordan, Kaplan, & Dick, 2001). For example, second-grade students with MD answered fewer number combinations correctly than students without MD (Hanich et al., 2001). A similar trend emerged

Continued on page 12

TABLE 1. Types of Number Combinations

Type	Terminology	Examples
Addition	addend + addend = sum	3 + 4 = 7 9 + 6 = 15
Subtraction	minuend - subtrahend = difference	4 - 1 = 3 13 - 8 = 5
Multiplication	factor x factor = product	3 x 2 = 6 8 x 7 = 56
Division	dividend ÷ divisor = quotient	9 ÷ 3 = 3 72 ÷ 8 = 9

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with third- and fourth-grade students where students with MD performed significantly lower and made more errors on a test of number combinations than students without MD (Andersson, 2008). These deficits in number combinations skill may stem from difficulties in storing and retrieving number combinations from long-term memory or from deficits in keeping number combinations in working memory (e.g., Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Jordan & Montani, 1997), as elaborated by Daniel Berch in this issue. Because past attempts at remediating number combinations have been equivocal (Kroesbergen & Van Luit, 2003), we conducted four studies to investigate how to help students develop fluency with number combinations skill.

Four Studies Designed to Improve Fluency with Number Combinations

To investigate ways to help students with MD with number combinations, we conducted four studies over a four-year span, one each year. Each study was carried out with third graders. The four studies, which focused on acquisition and fluency performance on addition and subtraction number combinations, constituted a program of research in which each study was designed to address questions that arose in the prior studies. In this way, a linear progression exists across the studies as we learned more each year about number combinations instruction and learning. See Figure 1 for a flowchart of the four studies. These four studies are described in greater depth in Fuchs, Powell, Seethaler, Fuchs, et al. (2010). Students in all four studies struggled with mathematics (i.e., performing below the 26th percentile on a standardized mathematics computation test of addition, subtraction, multiplication, and division) although their underlying sources of the mathematics difficulty may have varied. Students in all four studies were identified as having MD in the same manner with the same standardized test.

Study 1: Practice, practice, practice. The first iteration of number combinations tutoring included three daily activities: computer drill-and-practice, flash cards, and a review (Fuchs et al., 2008). With computer drill-and-practice, students were presented with number combinations for 7.5 minutes. A number combination "flashed" on a computer screen (e.g., 5 + 4 = __). It disappeared within a second or two; then the student typed in the entire combination (e.g., 5 + 4 = 9) from memory. As the student typed in the number combination, a number line picture filled in at the top of the screen. For addition problems, the first addend was represented in blue boxes and the second addend was represented in yellow boxes. For subtraction problems, the minuend boxes filled in with a yellow color, and the subtrahend was represented by Xs over the minuend boxes. If a student answered the number combination correctly, they heard applause and earned a point toward treasures that collected in a treasure box on the screen. If the student answered incorrectly, they were required to retype the combination until it was answered correctly. At the end of 7.5 minutes, the student's score (i.e., number of correctly answered combinations) was presented on the computer screen. After this computer drill-and-practice, students answered number combination flash cards, presented by the tutor, for 2 minutes. At the end of 2 minutes, the tutor and student counted the correctly answered flash cards and placed the score on a graph. After 3 consecutive sessions of

FIGURE 1. Progression of Study Designs for Four Studies on Number Combinations (NC)

Study 1					
		Computer drill-and-practice	NC flash cards/ Number line flash cards		Paper review
Study 2					
	Explicit instruction (NC)	Computer drill-and-practice	NC flash cards	Number line flash cards	Paper review
Study 3					
Counting strategies	Explicit instruction (word problems)	Computer drill-and-practice	NC flash cards		Paper review
Study 4					
Counting strategies	Explicit instruction (word problems)		NC flash cards		Paper review

answering 35 number combinations flash cards correctly, the tutor then switched to number line flash cards for the flash card activity. The number line flash cards represented combinations similar to those presented during computer drill-and-practice. Students were asked to state the combination represented by the number line. Similar to number combinations flash cards, students graphed the number of correctly-answered cards after 2 minutes. Each session concluded with a paper review of 15 number combinations. Students had 2 minutes to answer as many number combinations as possible.

Conditions. We compared this *number combinations* tutoring to three other tutoring conditions: (a) *double-digit computation and estimation*, (b) *double-digit computation and estimation with number combinations* tutoring, and (c) *word identification*. In double-digit computation and estimation tutoring, students worked through double-digit addition and subtraction problems via a computer program, answered double-digit flash cards, and answered double-digit problems on a paper review. Double-digit computation and estimation with number combinations tutoring included the components of both double-digit computation and estimation tutoring and number combinations tutoring. Word-identification tutoring students participated in computer drill-and-practice on sight words and read passages aloud for fluency practice; this was a control condition. All students received tutoring for 15 weeks, 3 sessions per week, 15–30 minutes per session. Tutoring sessions were delivered during the school day at times designated by the classroom teacher. Students were tutored individually in locations outside of the student's classroom (i.e., hallway, empty classroom, library, or conference room).

Results. (See Table 2 for results from the four studies. Effect sizes (ES) are reported for significant results.) Students participating in number combinations tutoring demonstrated significantly stronger improvement in number combinations compared to students in the double-digit computation and estimation condition (ES = 0.69), the double-digit and number combinations condition (ES = 0.81), or the word identification condition (ES = 0.78). From this first study, we learned that students who were tutored on number combinations alone demonstrated stronger improvement than those students who received number combinations tutoring in conjunction with tutoring on double-digit computation and estimation. We also learned that the combination of computer drill-and-practice, flash cards, and paper review appeared to improve fluency with number combinations.

Study 2: Providing conceptual instruction. To determine the importance of conceptual instruction for number combinations remediation, the second study compared the number combinations tutoring of Study 1 to an expanded number combinations tutoring with conceptual number combinations instruction (Powell, Fuchs, Fuchs, Cirino, & Fletcher, 2009). *Number combinations* tutoring included only three activities, which were similar to the number combinations activities in Study 1: computer drill-and-practice, number combinations flash cards, and paper review. The *expanded* tutoring included six activities: conceptual instruction, number combinations flash cards, computer drill-and-practice, number line flash cards, combinations family review, and paper review. (The two flash card activities, computer drill-and-practice, and paper review were similar to the activities described above.) With the conceptual instruction, tutors and students worked with manipulatives (i.e., blue and yellow blocks) to show various combinations of a fact family (i.e., 2 + 4 = 6; 4 + 2 = 6; 6 – 2 = 4; 6 – 4 = 2). Students then practiced generating families of number combinations on the combinations family review.

Conditions. We compared the performance of the students in the two number combinations conditions described above, referred to here as *number combinations*, and *expanded number combinations,* to students in two competing conditions: (a) *double-digit computation and estimation* tutoring and (b) no tutoring (i.e., control). The *double-digit computation and estimation* tutoring was similar to the tutoring described in Study 1. Tutoring for students in all three tutoring conditions lasted 15 weeks, 3 sessions per week, 15–25 minutes per session.

Continued on page 14

TABLE 2. Results from Number Combinations Studies

Study	Conditions	Significant results on NC	Effect sizes
1	Number combinations tutoring (NC) Double-digit computation and estimation tutoring (DD) Double-digit computation and estimation with number combinations tutoring (COMB) Word identification tutoring (CONTROL)	NC > DD NC > COMB NC > CONTROL	0.69 0.81 0.78
2	Number combinations tutoring (NC) Expanded Number combinations tutoring with conceptual instruction (E-NC-CONC) Double-digit computation and estimation tutoring (DD) No tutoring (CONTROL)	NC > DD NC > CONTROL E-NC-CONC > DD NC-CONC > CONTROL NC = NC-CONC	0.31 0.50 0.37 0.53 –
3	Number combinations tutoring with counting strategies (NC-COUNT) Word-problem tutoring with counting strategies (WP-COUNT) No tutoring (CONTROL)	NC-COUNT > CONTROL WP-COUNT > CONTROL NC-COUNT = WP-COUNT	0.52 0.62 –
4	Word-problem tutoring with counting strategies practice (WP-COUNT) Word-problem tutoring (WP) No tutoring (CONTROL)	WP-COUNT > CONTROL WP-COUNT > WP WP > CONTROL	0.67 0.22 0.43

Results. Students in both number combinations conditions performed similarly to one another. Students who received number combinations tutoring (without conceptual instruction) significantly outperformed students in the double-digit tutoring (ES = 0.31) and control (ES = 0.50) conditions. Similarly, students receiving the expanded number combinations tutoring demonstrated significantly higher scores than double-digit tutoring (ES = 0.37) and control students (ES = 0.53). It was interesting to note that students in either number combinations conditions performed similarly even though the students who received expanded tutoring on number combinations spent much more time on the conceptual basis of number combinations, having to work through and think about how number combinations relate to one another. This finding suggests that students with MD needed to learn a plan or strategy for solving number combinations when the answer is not immediately recalled. For this reason, we introduced counting strategies in Study 3.

Study 3: Counting strategies. Number combinations tutoring in Study 3 included instruction on how to use counting strategies to solve addition and subtraction combinations (Fuchs et al., 2009). (See Figure 2 for an explanation of the counting strategies.) Number combinations tutoring included five activities: 1) number combinations flash cards, 2) explicit instruction, 3) lesson-specific flash cards, 4) computer drill-and-practice, and 5) paper review. The number combination flash cards, computer drill-and-practice, and paper review used in this condition were the same as described in Studies 1 and 2. Explicit instruction focused on teaching and practicing the counting strategies along with instruction focused on groups (not families) of number combinations (e.g., the +0 and –0 combinations; the +1 and –1 combinations). The lesson-specific flash cards also focused on the groups; students answered these cards for 1 minute. As students moved on to the next group, they were permitted to take the lesson-specific flash cards home for practice.

Conditions. In addition to the *number combinations* tutoring condition used in this study, the comparison conditions in Study 3 were (a) *word-problem* tutoring and (b) no tutoring (i.e., control). Word-problem tutoring consisted of teaching students to read, set up, and solve word problems by problem type. Students also learned the counting strategies taught in number combinations tutoring. Students in each of the two tutoring conditions received instruction over 16 weeks, 3 sessions per week, 20–30 minutes per session.

Results. On number combinations, students participating in number combinations tutoring outperformed control students (ES = 0.52). There were no differences between students receiving number combinations or word-problem tutoring, given that word-problem tutoring also included instruction on solving number combinations with counting strategies. Interestingly, word-problem tutoring students outperformed control students on number combinations (ES = 0.62). From this study, we learned that explicit instruction and practice on using counting strategies to solve number combinations appeared to be an important component of number combinations instruction.

Study 4: Daily practice with counting strategies. Because all tutored students, regardless of whether counting strategies and number combinations were the focus of tutoring, improved reliably more than control students on number combinations in Study 3, the goal of Study 4 was to determine how much practice on counting strategies was necessary (Fuchs, Powell, Seethaler, Cirino et al., 2010). In Study 4, students in each of two active tutoring conditions received instruction on word problems. The nature of word-problem instruction in the two conditions was identical. The only difference between the two conditions was that only one condition included daily practice on number combinations, and the other did not. For instance, in both conditions, students received an initial explanation lesson on counting strategies, but in only one condition did students also receive daily practice on solving number combinations with counting strategies. Specifically, in both conditions, students participated in word-problem warm up, explicit word-problem instruction, word-problem sorting cards, and paper review. These activities focused on solving word problems belonging to three word-problem types: total, difference, and change. Students in both conditions also participated in flash cards, but the nature of the flash card activity differed across the two conditions. Children in the word problem tutoring without the practice condition simply read numbers on flash cards. Children in the number combinations instruction with daily practice condition answered number combinations flash cards for 1 minute. If a student answered incorrectly, the tutor asked him or her to use a counting strategy until the student answered correctly. At the end of 1 minute the number of correctly answered flash cards was recorded on a graph. During counting up practice, the tutor asked the student to solve four number combinations using counting strategies. In summary, although students in the word-problem tutoring without daily practice received explicit instruction on using counting strategies to solve number combinations, they did not participate in the number combinations flash cards or counting practice each session.

Conditions and results. Tutoring in both active conditions lasted 16 weeks, 3 sessions per week, 20–30 minutes per session. We also incorporated a no tutoring condition (i.e., control). On number combinations, students who received

FIGURE 2. Counting Strategies

COUNTING UP Addition	COUNTING UP Subtraction
1. Put the bigger number in your fist and say it. 2. Count up the smaller number on your fingers. 3. Your answer is the last number you say.	1. Put the minus number in your fist and say it. 2. Count up your fingers to the number you start with. 3. Your answer is the number of fingers you have up.

word-problem tutoring *with* daily counting strategies practice outperformed students in the control condition (ES = 0.67) and students who received the word-problem tutoring *without* daily counting strategies practice condition (ES = 0.22). Nevertheless, students who received word-problem tutoring without daily practice outperformed control students (ES = 0.43). From Study 4, we learned that students require daily practice on number combinations even if the time spent on number combinations is only a few minutes per session.

Lessons Learned Across the Four Studies

In Studies 1 and 2, we provided third graders with tutoring focused wholly on number combinations. The instruction included a variety of activities ranging from flash cards to computer drill-and-practice to explicit conceptual instruction to paper reviews. We hoped that a package of diverse number combinations activities would be the best route for remediation of number combinations skill. In Studies 3 and 4, however, we provided brief but explicit instruction on solving number combinations, and we learned that students benefit from learning a strategy to solve number combinations. Students benefitted from counting strategies instruction embedded within word-problem tutoring just as much as tutoring programs focused exclusively on number combinations. So, we learned that number combinations instruction can be provided alongside instruction on word problems, and students still derive benefit on number combinations.

Starting in Study 3, we began to provide students with counting strategies for solving number combinations when they did not automatically recall the answer (i.e., counting strategies). We felt students needed a reliable and efficient backup plan. In Study 4, we isolated the effect of practice within counting strategies instruction and our findings suggest the importance of daily (albeit brief) practice for this population of learners. In this way, we conclude that providing students with explicit instruction (i.e., counting strategies) on solving number combinations and practicing these counting strategies is an important and efficient component of mathematics instruction for students with MD.

Additionally, we note that, across studies, students participated in daily flash card practice and a paper review to improve number combinations fluency. Although we never isolated the contribution of flash card practice or paper reviews, we believe that reviewing number combinations and requiring students to use counting strategies to solve flash cards answered incorrectly and graphing the flash card score, may be another important component of number combinations instruction. Along the same lines, reviewing number combinations on paper may be important for ensuring transfer to paper-and-pencil tasks.

Will All Students with MD Benefit from Number Combinations Instruction?

As already mentioned, in each of the four studies in this program of research, all participants were third grade students who struggled with mathematics. Some of these students also struggled with reading difficulties (RD) (i.e., scored below the 26th percentile on a standardized reading test) and were therefore categorized as having MD+RD. Other students performed above the 40th percentile in reading and were therefore categorized as having MD-only. In each of the four studies, we looked for performance differences between MD+RD and MD-only students. Some prior research has shown performance differences on number combinations between students with MD+RD and MD-only students (Hanich et al., 2001) with MD-only students outperforming MD+RD students, whereas other research has not demonstrated differences between students with MD+RD and MD-only on number combinations (Geary, Hoard, & Hamson, 1999).

Unfortunately, in all but one of our four studies reviewed here, the number of students in each subgroup was too small to determine whether response to intervention differed as a function of whether students experienced MD alone or in combination with RD. Yet, some patterns in the data provide the basis for hypothesizing that relative to students with MD-only, students with MD+RD may require more intensive intervention, and different kinds of intervention that have more systematic practice with a greater emphasis on language. Large-scale intervention research is, however, needed—with adequately large samples of students with MD with and without RD—to assess the tenability of these hypotheses.

Advice for Teachers and Parents

Based on this program of research on number combinations remediation, many students with MD benefit from explicit instruction on how to solve number combinations. Students should be provided with strategies for solving combinations when the answer is not immediately recalled and should receive many opportunities to practice solving number combinations through a variety of formats (i.e., flash cards, oral quizzes, paper reviews). From what we have learned from our program of research, we feel that the more opportunities students have to see and work with number combinations, the more likely they will improve their number combinations skill. More opportunities does not necessarily mean more *time* practicing number combinations, because, as we have learned in our studies, it appears that students benefit from well-designed and implemented number combinations tutoring that lasts a few minutes each session just as much as number combinations instruction that lasts 30 minutes each session. We believe students should be provided with explicit instruction on solving number combinations and provided with different outlets to practice number combinations throughout the school year.

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Continued on page 16

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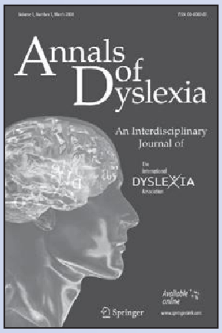

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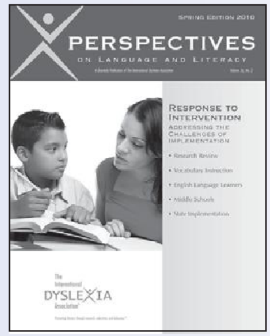

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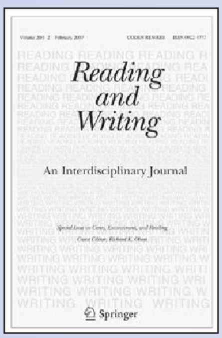

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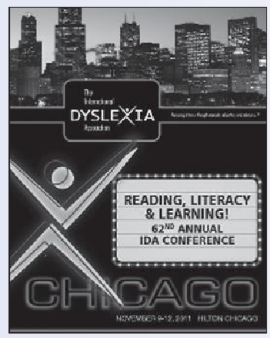

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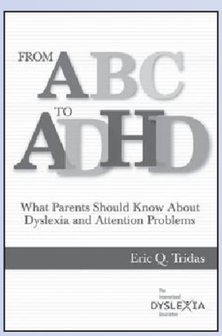

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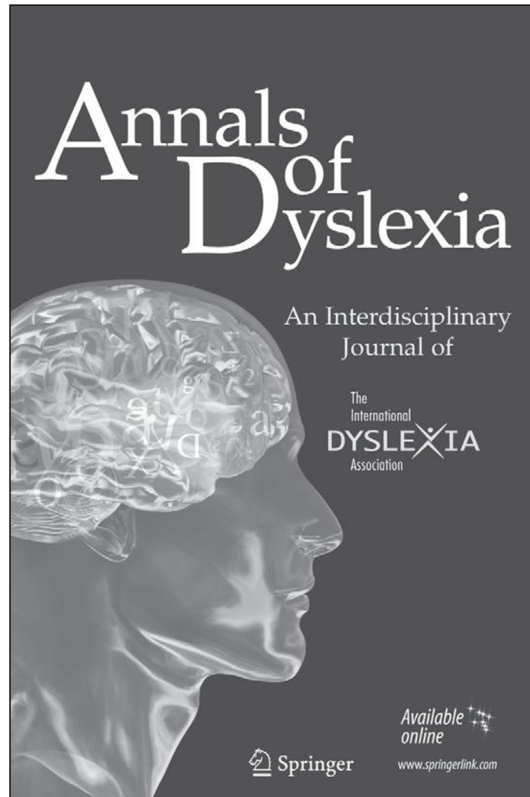

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Working Memory Limitations in Mathematics Learning: Their Development, Assessment, and Remediation

by Daniel B. Berch

Some school-age children struggle with mathematics, routinely experiencing difficulty in learning or remembering basic arithmetic facts and carrying out even the seemingly most elementary numerical operations (Berch & Mazzocco, 2007). Such difficulties are compounded when students are expected to build upon these basic skills as they are introduced to increasingly abstract, mathematical content domains. Consider a letter published in the *Washington Post* written by a seventh-grade teacher not that long ago:

> Many of the seventh graders I teach have a poor sense of numbers. They don't understand that adding two numbers results in a larger number, that multiplication is repeated addition, that 5 × 6 is larger than 5 × 4 or that one quarter is smaller than one half. This lack of basic math facts detracts from their ability to focus on the more abstract operations required in math at a higher level" (Susan B. Sheridan, *Washington Post*, December 27, 2004).

What are the key factors contributing to this state of affairs? Is the problem due primarily to poor instruction, or is there something inherently difficult about learning even basic arithmetic because of the ways in which the developing child's mind works? Have we been able to trace the origins of extremely low math performance that would warrant the diagnosis of a mathematical learning disability? And do effective remedial approaches exist for improving the mathematics achievement of such children?

As it turns out, definitive answers to these weighty questions still elude us. Nonetheless, progress is being made on a number of fronts, especially in the study of the fundamental cognitive processes that underlie mathematical thinking in general and those that are crucial for achieving proficiency in carrying out arithmetic calculations in particular. In this article, I will review what we have learned about the contributions of an especially important factor known as "working memory," along with the difficulties that can arise for students who exhibit weaknesses if not outright deficits in the full complement of skills comprising this construct.

Introduction to the Concept of Working Memory

Precisely what do we mean when invoking the concept of working memory? As this cognitive construct actually encompasses several mental operations, definitions of working memory tend to vary considerably (Dowker, 2005; Shah & Miyake, 1999). Furthermore, although this concept seems comparatively straightforward at one level, it turns out to be much more complicated at another. Such a view is shared by many, including Susan Pickering, a leading researcher in this field who acknowledged that "The concept of working memory is both reassuringly simple and frustratingly complex" (2006, p. xvi).

As a consequence, it may prove instructive to present an example of how working memory can influence arithmetic problem solving before providing a definition. To begin with, consider the following quote taken from Lewis Carroll's *Through the Looking-Glass* (1871) which Kaufman (2010) describes as "A working memory lapse in Wonderland" (p. 153): "'And you do addition?' the White Queen asked. 'What's one and one and one and one and one and one and one and one and one and one?' 'I don't know,' said Alice, 'I lost count.'"

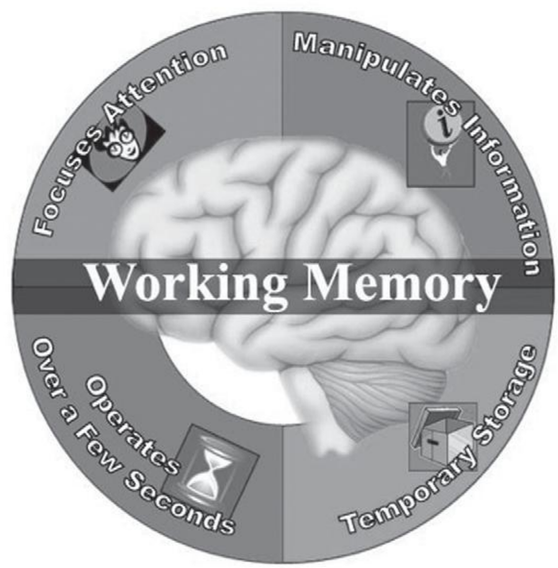

Figure 1. Graphic illustration of the defining features and components of the working memory system. Reprinted from Working-Memory-and-Education – Introduction to Working Memory (WM), D. B. Berch, Retrieved November 17, 2010, from http://working-memory-and-education.wikispaces.com/Introduction+to+Working+Memory+(WM). Copyright 2009 by Carren Tatton. Reprinted with permission.

Although it is doubtful that Alice's failure to solve this problem is attributable to a mathematical learning disability, the example illustrates nicely some of the key components of working memory depicted in Figure 1. That is, in order not to lose count when attempting to solve such a problem, an individual would have to: a) *focus attention* on each new addend as it is presented, b) *manipulate the information* by mentally adding the "ones," and at the same time, c) selectively maintain some of the information (in this case, the most recent prior sum)

Continued on page 22

in *temporary mental storage*, and d) complete all of these tasks within the span of *a few seconds*. In other words, working memory is probably best defined as a limited capacity system responsible for temporarily storing, maintaining, and mentally manipulating information over brief time periods to serve other ongoing cognitive activities and operations. In essence, it constitutes the mind's workspace.

Getting back to the White Queen's arithmetic problem, while adding single digits should be comparatively easy for most typically achieving seven-and-a-half-year-olds (Alice's age), it is evident from this example that one can excessively tax working memory by requiring a learner to simultaneously attend, store, and mentally process a rather large amount of information (albeit elementary in some sense) within a relatively short period of time. As Susan Gathercole, another leading researcher in this field has pointed out, overloading this fragile mental workspace can lead to "complete and catastrophic loss of information from working memory" (Gathercole, 2008, p. 382).

. . . working memory is probably best defined as a limited capacity system responsible for temporarily storing, maintaining, and mentally manipulating information over brief time periods to serve other ongoing cognitive activities and operations.

Obviously, no teacher would deliberately choose to overload his or her students' working memory capacity. Nevertheless, mathematical information can sometimes be presented in a manner (e.g., orally or in textbooks) that inadvertently strains the processing capacity of students. Practitioners can learn to readily avoid these situations if they are furnished with some basic information about the nature of working memory, its limitations, and the ways in which students can differ with respect to its constituent skills. Accordingly, the purpose of this article is to provide non-specialists with a succinct overview of the latest research on this topic, which I have organized in a way that I hope will shed light on some of the most important questions pertaining to the role of working memory in learning school mathematics, including: What are the ways in which working memory's component skills can be measured? How do limitations in working memory contribute to the development of mathematical learning difficulties and disabilities? And finally, what kinds of instructional interventions or remedial approaches are available for mitigating the detrimental effects of working memory limitations on mathematics achievement?

How Are Working Memory Skills Measured?

Children's working memory skills are customarily assessed with a variety of what are referred to as "simple" and "complex" span tasks. Simple span tasks are used to measure the short-term storage capacity of two types of *domain-specific* representations: verbal and visuospatial. To appraise the former, a reading or listening span measure is usually employed that entails the recall of word or number sequences; when assessing the latter, either the recall of random block-tapping sequences or randomly filled cells in a visual matrix or grid is typically required.

In contrast, complex span tasks gauge *domain-general*, central attentional resources by imposing substantial demands both on mental storage and processing (Holmes, Gathercole, & Dunning, 2010). As I have described elsewhere (Berch, 2008), a particularly representative example of such a measure is the Backward Digit Span task in which a random string of number words is spoken by the examiner (e.g., saying "seven, two, five, eight . . ."), and the child must try to repeat the sequence in reverse order. Note that rather than *simply* having to recall the numbers in the same forward order (which is considered a measure of the short-term, verbal storage component per se), the backward span task requires that the child both store and maintain the forward order (i.e., verbal component) of the number words while simultaneously having to mentally manipulate this information to accurately recite the original sequence in the opposite order. It is this dynamic coordination and control of attention combined with the storing and manipulation of information in support of ongoing cognitive activities that I characterized earlier as being the *sine qua non* of working memory.

To carry out a comprehensive assessment of children's working memory capacities, most researchers currently make use of one of two standardized batteries—the *Working Memory Test Battery for Children* (Pickering & Gathercole, 2001) or the *Automated Working Memory Assessment* (Alloway, 2007). As Holmes and her colleagues (2010) describe, each of these is comprised of several subtests, affording multiple assessments of different facets of working memory (e.g., central attentional resources as well as verbal and visuospatial short-term storage components). Additionally, these batteries permit the identification of children with poor working memory for their chronological age, based on existing norms.

Another technique for identifying children with poor working memory is derived from ratings provided by a child's teacher, a prominent example being the *Working Memory Rating Scale* (Alloway, Gathercole, & Kirkwood, 2008). This measure consists of approximately 20 statements of problem behaviors such as "She lost her place in a task with multiple steps" and "The child raised his hand but when called upon, he had forgotten his response." Teachers rate how typical each of these behaviors is of a given child using a four-point scale. Although this technique affords a fast and efficient method for initial identification of working memory problems in a school setting (Holmes et al., 2010), it is probably best used as one component of a comprehensive evaluation by the school psychologist. Furthermore, if need be, teachers can choose to make supplementary, informal observations for guiding adjustments to their instructional approaches with particular children.

How Do Working Memory Limitations Contribute to Mathematical Learning Difficulties?

As noted earlier, measures of working memory are usually designed to assess one or more of three presumed subsystems comprising what is known as a multicomponent model: a domain-general, limited capacity central executive that governs the storage and temporary maintenance of information in two domain-specific representational subsystems—the phonological loop and visuospatial sketchpad—by means of attentional control (Baddeley, 1990, 1996; Baddeley & Hitch, 1974). To date, the vast majority of investigations aimed at determining particular relationships between various working memory skills and mathematics learning or performance have been based on this model.

Such relationships have been studied in children ranging from preschool age to adolescence, and for math skills extending from the very basic (e.g., numerical transcoding—writing an Arabic numeral in response to hearing a number word, counting, numerical magnitude comparison, and single-digit addition and subtraction) to more complex mathematical operations and content domains, such as multidigit arithmetic, rational numbers, and algebraic word problem solving. Furthermore, according to Raghubar, Barnes, and Hecht (2010), numerous other factors may influence and therefore complicate the interpretation of findings pertaining to the relations between working memory and math performance, including but not limited to skill level, language of instruction, how math problems are presented, the type of math skill at issue, whether that skill is just being acquired or has already been mastered, the type of working memory task administered, and the kinds of strategies that different-aged children operating at diverse skill levels may employ for a given task.

Consistent with this perspective, Geary and his colleagues (Meyer, Salimpoor, Wu, Geary, & Menon 2010) highlighted the importance of their findings that the contributions of particular components of working memory to individual differences in mathematics achievement can vary with grade level or the type of math content being assessed. More specifically, these researchers showed that the central executive and phonological loop play a more important role in facilitating mathematics performance for second graders, while the visuospatial sketchpad does so for third graders. Furthermore, they provide a compelling argument that this grade-level difference is attributable to instruction and practice rather than a developmental change in working memory capacity.

All this being said, earlier reviews of research on this topic (DeStefano & LeFevre, 2004; Swanson & Jerman, 2006) along with more recent ones (Geary, 2010; Raghubar et al., 2010) have yielded reasonably clear evidence of a generally strong association between working memory capacity and mathematics performance. Indeed, even the leading proponent of the view that the development of mathematical learning disabilities is attributable to a deficit in a domain-specific, inherited system for coding the number of objects in a set has recently acknowledged that the domain-general, central executive functions of working memory are at the very least associated (i.e., correlated) with arithmetic learning and performance (Butterworth, 2010). What is the nature of this relationship? As Geary (2010) concludes after reviewing the findings, the greater the capacity of the central executive, the better the performance both on cognitive mathematics tasks and math achievement tests (Bull, Espy, & Wiebe, 2008; Mazzocco & Kover, 2007; Passolunghi, Vercelloni, & Schadee, 2007). Furthermore, Geary notes that the phonological loop seems to be important for verbalizing numbers, as in counting (Krajewski & Schneider, 2009) and in solving math word problems (Swanson & Sachse-Lee, 2001).

. . . factors (that) may influence . . . the relations between working memory and math performance (include) skill level, language of instruction, how math problems are presented, the type of math skill at issue, whether that skill is just being acquired or has already been mastered, the type of working memory task administered, and the kinds of strategies that different-aged children operating at diverse skill levels may employ for a given task.

Although studies have also shown that children with either math learning difficulties or disabilities exhibit deficits in all three working memory subsystems, Geary (2010) concludes that impairment in their central executive appears to be particularly troublesome (Bull, Johnston, & Roy, 1999; Swanson, 1993). However, Geary also observes that the interpretation of these findings is complicated by the fact that at least three purported subcomponents of the central executive (i.e., inhibition, updating, and attention shifting) have been found to influence math learning in different ways (Bull & Scerif, 2001; Murphy, Mazzocco, Hanich, & Early, 2007; Passolunghi, Cornoldi, & De Liberto, 1999; Passolunghi & Siegel, 2004).

In summing up what researchers have learned about associations between working memory and math learning disabilities, Geary (2010) affirms that: "At this point, we can conclude that children with MLD have pervasive deficits across all of the working memory systems that have been assessed, but our understanding of the relations between specific components of working memory and specific mathematical cognition deficits is in its infancy" (p. 62).

What Kinds of Interventions or Remedial Approaches Exist for Improving Working Memory?

In a review of techniques used to date for mitigating the difficulties encountered by children who have poor working memory, Holmes and her colleagues (2010) grouped these under three main approaches: 1) a classroom-based intervention that consists of encouraging teachers to adapt their instructional approaches in ways that minimize working memory loads; 2) training designed to teach children to make use of

Continued on page 24

 continued from page 23

TABLE 1. Principles of the Classroom-Based Working Memory Approach

Principles	Further Information
Recognize working memory failures	Warning signs include recall, failure to follow instructions, place-keeping errors, and task abandonment
Monitor the child	Look out for warning signs, and ask the child
Evaluate working memory loads	Heavy loads caused by lengthy sequences, unfamiliar and meaningless content, and demanding mental processing activities
Reduce working memory loads	Reduce the amount of material to be remembered, increase the meaningfulness and familiarity of the material, simplify mental processing, and restructure complex tasks
Repeat important information	Repetition can be supplied by teachers or fellow pupils nominated as memory guides
Encourage use of memory aids	These include wall charts and posters, useful spellings, personalized dictionaries, cubes, counters, abaci, Unifix blocks, number lines, multiplication grids, calculators, memory cards, audio recorders, and computer software
Develop the child's own strategies	These include asking for help, rehearsal, note-taking, use of long-term memory, and place-keeping and organizational strategies

Note. Adapted from "Working memory in the classroom," by S. E. Gathercole, 2008, *The Psychologist, 21*, 382–385. Copyright 2008 by The British Psychological Society. Adapted with permission.

memory strategies for improving the efficiency of working memory; and 3) training aimed directly at improving working memory through the use of an adaptive computerized program that involves repeated practice on working memory tasks.

The classroom-based intervention is founded on a set of seven principles that originated from both classroom practice and cognitive theory (Gathercole, 2008) and are summarized in Table 1. Recently, a research team carried out an evaluation over a one-year period of two forms of this intervention aimed at primary school children with poor working memory (Elliott, Gathercole, Alloway, Holmes, & Kirkwood, 2010). Although there was no evidence that the intervention programs directly improved either working memory or academic performance, the extent to which teachers implemented these seven principles was predictive of their students' mathematical (and literacy) skills. Furthermore, teachers were reportedly very pleased about the ways in which the intervention had improved their own understanding and practice (which exemplifies the kind of mathematics knowledge enhancement that Dr. Murphy and her colleagues (this issue) promote for all teachers). Additional studies exploring how best to implement this kind of intervention are clearly warranted if we are to determine the optimal ways for practitioners to enhance children's mathematics achievement through the strengthening of working memory skills.

With respect to the strategy training approach, the kinds of memory strategies children have been taught to use include repetitively rehearsing information, generating sentences from words or making up stories based on them, or creating visual images of the information (Holmes et al., 2010). Strategy training incorporating all of these techniques was recently administered to children ranging in age from five to eight years old in two sessions per week over a six-to-eight-week period using a computerized adventure game (St. Clair-Thompson, Stevens, Hunt, & Bolder, 2010). Although training significantly enhanced both verbal short-term memory and working memory, there were no gains in visuospatial short-term memory. More relevant to the focus of this article, performance on a mental arithmetic task improved significantly. Furthermore, all of these gains were evidenced by children with poor working memory as well as those with average working memory. Nevertheless, no significant changes emerged on standardized assessments of arithmetic or other mathematical domains either immediately following training or five months afterward.

. . . the extent to which teachers implemented these seven principles (of working memory intervention) was predictive of their students' mathematical (and literacy) skills.

Finally, according to Holmes and her colleagues (2010), the most impressive gains in working memory obtained thus far have resulted from a direct training program developed originally for use with children with attention deficit hyperactivity disorder (ADHD; Klingberg et al., 2005; Klingberg, Forssberg, & Westerberg, 2002). Children undergoing this intensive training regimen participate in a variety of computerized tasks designed to repeatedly tax their working memory capacity (i.e., requiring simultaneous storage and manipulation of information) to the greatest extent possible without exceeding a level they can still manage effectively. This is achieved by matching the difficulty of each successive task to a child's current memory span on a trial-by-trial basis. Holmes, Gathercole, and Dunning (2009) administered this so-called adaptive training program to 9- and 10-year-olds with poor working memory skills in 20 training sessions, each 35 minutes long, over a period of five to seven weeks. Not only did the children exhibit sizeable improvements in verbal and visuospatial working memory, but six months later these gains had still not declined. And even though no gains were found on a standardized mathematics reasoning test immediately after training, a small but significant improvement emerged on the six-month follow-up assessment.

In sum, although these three types of interventions have been shown to improve working memory skills, evidence of their impact on academic performance in general and on mathematics abilities in particular is as yet rather limited (Holmes et al., 2010). However, it is our hope that continued study of ways to enhance such outcomes will yield stronger proof regarding whether such training can transfer to students' mathematics performance.

One final investigation is worth describing here, primarily because even though it was a cognitive laboratory study, it has important implications for improving classroom instruction. Briefly, this investigation revealed that although the working memory capacity of seven-year-olds is smaller than that of older children and adults, their attentional processes are just as efficient—so long as their smaller working memory capacity is not exceeded (Cowan, Morey, AuBuchon, Zwilling, & Gilchrist (2010). However, when their working memory is overloaded, attentional efficiency declines, suggesting that interventions aimed at enhancing working memory will in turn improve attentional efficiency. As these researchers put it, "In general, children's attention to relevant information can be improved by minimizing irrelevant objects or information cluttering working memory" (p. 131).

Conclusions

Taken together, the research reviewed in this article shows that we are making significant progress toward achieving a more complete understanding of the nature of working memory, its typical course of development, and the best methods for assessing its various features. We have also made important advances in discerning how working memory limitations and impairments can hinder the attainment of proficiency in mathematics, and we have just begun to explore the most promising strategies that can be implemented to enhance the working memory skills most relevant for improving students' mathematical learning and performance. Finally, I hope that the information provided here will be of some use to those of you who teach in identifying working memory limitations in your students, modifying the instructional environment to minimize extraneous or distracting information that might interfere with efficient selective attention, and designing strategies for enhancing your students' working memory skills.

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Continued on page 26

 continued from page 25

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Why Can't Students Get the Concept of Math?

by Julie L. Booth

Though there are a number of reasons why students may have difficulty in mathematics at different points in development, one concern that can affect the learning of all students (regardless of whether they have a mathematics learning disability, or MLD) is a lack of conceptual understanding. The National Council of Teachers of Mathematics (2000) stresses the importance of conceptual understanding for learning in math and recommends alignment of facts and procedures with concepts to improve student learning. More recently, the National Mathematics Advisory Panel (2008) recommended helping students master both concepts and skills and maintained that preparation for algebra requires simultaneous development of conceptual understanding and computational fluency, as well as cultivation of students' skill at solving problems. As an indicator of the level of emphasis placed on conceptual understanding, the final report of the National Mathematics Advisory Panel (2008) uses the words "concept" or "conceptual" 87 times in 120 pages; in comparison, the word "procedure" or "procedural" is used fewer than 40 times.

. . . procedural knowledge can be operationally defined as* how to do *something, and conceptual knowledge . . . allows one to understand* why the *procedure is appropriate for that task.

Thus, there is consistent recommendation that teachers focus on concepts in mathematics. But what does it mean to focus on concepts, and how can this be done in a way that develops students' conceptual understanding without sacrificing attention to required procedural skills? To accomplish this goal, teachers must first understand what conceptual and procedural knowledge are, how these forms of knowledge differ from each other, and the relations among the two types of knowledge. The following sections focus on the domain of algebra, discussing the definitions of conceptual and procedural knowledge, describing some of the particular conceptual difficulties students tend to have and how these difficulties affect performance and learning, and presenting empirically-based solutions for how to effectively address the issues of conceptual understanding in real-world classroom settings.

Conceptual Versus Procedural Knowledge in Mathematics

Conceptual knowledge has been defined as "an integrated and functional grasp of mathematical ideas" (National Research Council, 2001, p. 118). Consistent with this and other research on learning in mathematics, conceptual knowledge can be viewed as recognizing and understanding the important principles or features of a domain as well as interrelations or connections between different pieces of knowledge in the domain (Hiebert & Wearne, 1996). In contrast, procedural knowledge is the ability to carry out a series of actions to solve a problem (Rittle-Johnson, Siegler, & Alibali, 2001). In short, procedural knowledge can be operationally defined as *how* to do something, and conceptual knowledge as an understanding of *what* features in the task mean; conceptual knowledge of those features collectively allows one to understand *why* the procedure is appropriate for that task.

Though conceptual and procedural knowledge are often discussed as distinct entities, they do not develop independently in mathematics and, in fact, lie on a continuum, which often makes them hard to distinguish (Star, 2005). This may be especially difficult in algebra, where many new procedures are taught over the course of the year (e.g., solving equations, factoring, graphing lines, etc.). Given the nature of the content in algebra courses, items designed to measure conceptual knowledge may have elements that resemble procedural tasks. However, the information extracted about students' knowledge is not about their ability to carry out procedures. For example, one could give students the graph of a line and ask them to find the slope (procedural knowledge), or one could give students the same graph and ask them how the slope would change if the x and y intercepts were reversed (conceptual knowledge). Similarly, one could provide a pair of fractions and ask students to add them (procedural knowledge), or one could ask students to compare the sizes of the fractions and think about what would happen if the numerators and denominators were reversed (conceptual). Furthermore, one could show students an algebraic equation and ask them to solve it (procedural knowledge), or one could ask whether that equation is equivalent (or has the same solution set) to another equation (conceptual knowledge). Thus, even with the same stimulus for a problem, one can acquire very different types of information about what students know by the way that one asks the students to think about the problem.

Conceptual Difficulties in Algebra

For the past few decades, researchers in the fields of cognitive development and mathematics education have maintained that students beginning algebra do not fully understand important concepts that teachers may expect them to have mastered from their elementary math and pre-algebra courses. Within the domain of equation solving alone, a number of concerning misconceptions have been identified, including that students believe that the equals sign is an indicator of operations to be performed (Baroody & Ginsburg, 1983), that negative signs represent only the subtraction operation and do not modify terms (Vlassis, 2004), that subtraction is commutative (Warren, 2003), and that variables cannot take on multiple values (Knuth, Stephens, McNeil, & Alibali, 2006).

Continued on page 32

(See Figure 1 for examples of student misconceptions.) Unfortunately, for many students, these misconceptions persist even after traditional classroom instruction on the relevant topic (Booth, Koedinger, & Siegler, 2007).

How do these strange conceptions develop, and why are they so persistent? One reason is that students who are struggling in a domain may not see lessons in that content area the way the teacher intends (Wenger, 1987). In a recent study, beginning algebra students (12–14 years old) were given a reconstruction task to assess their encoding of presented equations (Booth & Davenport, in preparation). In this task, individual equations were presented on a computer screen for 6 seconds. After an equation disappeared, students were asked to reconstruct it on paper, and responses were coded for the number and types of mistakes that students made in their reconstruction. Results showed that students who have poor conceptual knowledge encode presented equations less effectively than their high-knowledge peers. Further, students with misconceptions about specific features were most likely to make errors on those features. For example, consider the equation $4x = 9 + 7x - 6$. Students who do not think that negative signs are attached to the terms they modify often make errors such as displacing a negative sign ($4x = 9 - 7x + 6$) or deleting the negative sign ($4x = 9 + 7x + 6$); students who hold misconceptions about the equals sign make errors such as moving the equals sign ($4x + 9 + 7x = 6$) or inserting an additional equals sign ($4x = 9 = 7x + 6$).

FIGURE 1. Student Misconceptions About Three Key Algebraic Concepts: A) The Equals Sign, B) Negative Numbers, and C) Variables and Like Terms

A) Equals Sign	3 + 4 = 7 What is the name of this symbol? equal sign What does the symbol mean? so you know where to put the answer. Can the symbol mean anything else? If yes, please explain. Not Not really
B) Negative numbers	State whether each of the following is equal to -4x + 3: a. 4x + 3 Yes No b. 3 – 4x Yes No c. 4x – 3 Yes No d. 3 + (-4x) Yes No e. 3 + 4x Yes No
C) Variables/ Like Terms	State whether each of the following is an effective first step for simplifying 2d + 7 + 5: a. Combine 2d and 7 Yes No b. Combine 2d and 5 Yes No c. Combine 7 and 5 Yes No d. Combine 2d, 7, and 5 Yes No

In A), the student correctly names the equals sign, but then indicates that its only meaning is to show you where the answer goes. In B), the student correctly endorses items *b* and *d*, and correctly rejects *a* and *e*, but incorrectly endorses 4x – 3 as equivalent to -4x + 3. In C), the student indicates that combining any two of the terms is an acceptable first step, even though the terms given in a and b are not like terms (in both cases, one is a variable, and one is a constant).

. . . one can acquire very different types of information about what students know by the way that one asks the students to think about (a math) problem.

Another reason is that these misconceptions may have been ingrained in students due to particularities in the nature of their arithmetic instruction (Baroody & Ginsburg, 1983). For example, the misconception that the equals sign indicates where the answer goes is likely due, at least in part, to the way math facts and early addition problems are presented by teachers and in textbooks. Such problems are often presented vertically, with one number on top of the other, and then a solid line between the addends and the answer. When students are given horizontally presented problems, they are typically in a format such as 4 + 5 = 9, with numbers and operations appearing to the left side of the equals sign, and the answer (or a blank space for the answer) on the right side; students are rarely, if ever, exposed to other formats such as 9 = 4 + 5 (Seo & Ginsburg, 2003). McNeil (2008) found that even having students practice simple arithmetic problems in the typical format (4 + 5 = 9) as opposed to non-standard presentations (28 = 28) increased failure at mathematical equivalence problems (e.g., 3 + 5 + 6 = ___ + 6). Just imagine how much exposure to misleading problem formats students have gotten before they reach their Algebra 1 class, and how that might prompt them to approach algebraic equations!

The Danger of Misconceptions for Performance and Learning in Mathematics

As you might predict, these types of misconceptions are detrimental to students' *performance* on equation-solving tasks: students who hold misconceptions about critical features in algebraic equations solve fewer problems correctly (Booth & Koedinger, 2008). Even more interesting, these misconceptions are associated with the use of particular, related, but incorrect strategies when students attempt to solve problems. For example, students who do not think of negative signs as connected in any way to the subsequent numerical term often delete or move negatives within equations or subtract a term from both sides of the equation to eliminate the term even when the value in question is already negative; similarly, students who do not think of the equals sign as an indicator of balance between the terms on either side often delete or move the equals sign, or perform operations to only one side of the equation (Booth & Koedinger, 2008).

More crucially, such misconceptions also hinder students' *learning* of new material. Students who begin an equation-solving lesson with misconceptions learn less from a typical algebra lesson than students with more sound conceptual knowledge (Booth & Koedinger, 2008). Why might this be the case? One reason is highly related to abundant research in science education which demonstrates the importance of engaging and correcting students' preconceptions about scientific topics before presenting new information (Brown, 1992). If these preconceptions are not engaged, teachers are just attempting to pile more information on top of a flawed foundation built on persistent misconceptions. In this case, students will not achieve full comprehension of the new material (Kendeou & van den Broek, 2005); rather, they may reject the new information that does not fit with their prior conception or try in vain to integrate the new information into their flawed or immature conceptions, resulting in a confused understanding of the content (Linn & Eylon, 2006). Further, recall that struggling students may not correctly encode the features of the equations they are presented by their teacher and their textbook (e.g., Booth & Davenport, in preparation). How can students be expected to learn what the teacher intends if they are not correctly viewing, let alone interpreting, the instructional materials? Eliminating student misconceptions should be a critical goal for successful mathematics instruction, as discussed further by Dr. Murphy and her colleagues elsewhere in this issue.

Advantages of Strong Conceptual Knowledge for Learning in Mathematics

It is clear that reduction of misconceptions could have a great impact on student learning in mathematics, potentially allowing students to perform at a grade-appropriate level. However, eliminating misconceptions only takes students halfway toward the instructional goal of cultivating strong conceptual understanding.

Research in a wide variety of domains has identified a number of advantages of strong conceptual understanding, including having an easier time recalling new information (Chi, 1978), better categorization of new information (Chi, Feltovich, & Glaser, 1981), improved use and acquisition of problem-solving strategies (Gaultney, 1995), improved reasoning skills when interpreting new information (Gobbo & Chi, 1986), and a greater likelihood of inferring information that was not explicitly present in the instruction (Chi, Hutchinson, & Robin, 1989). These advantages have been attributed to more focused attention to goal-relevant features when representing and solving problems (Chi et al., 1981), quick encoding of information which is easily stored in and retrieved from long-term memory (Ericsson & Kintsch, 1995), and constraints on ranges of possible responses, leading to greater endorsement of reasonable incorrect responses rather than unreasonable ones (Ornstein, Merritt, Baker-Ward, Furtado, Gordon, & Principe, 1998). In other words, students with strong conceptual knowledge about a topic are likely to continue to learn more because their prior knowledge makes it easier for them to process and use new information related to that topic.

Though not all of these advantages have been investigated within the domain of mathematics, there is evidence that strong conceptual understanding of mathematics concepts leads to greater learning in mathematics. For example, Griffin, Case, and Siegler (1994) found that providing low-income children with a strong knowledge base of mathematical fundamentals helped those children to learn early arithmetic; students with a stronger understanding of numerical systems are also able to learn more about arithmetic (Case & Okamoto, 1996). Similarly, stronger pretest conceptual knowledge in algebra predicted students' learning to solve algebraic equations, above and beyond that predicted by more general math achievement (Booth & Koedinger, 2008). More research may be necessary to investigate the prevalence and nature of such advantages in math; however, it stands to reason that to give our students the best possible chance of success in mathematics, we must both eliminate student misconceptions and fill those gaps with a strong foundation in mathematical concepts.

Cultivating Strong, Correct Conceptual Understanding in the Classroom

What is being suggested may seem like a tall order: Teachers need to target misconceptions and build up students' conceptual knowledge, all while still providing students with enough instruction and practice on the wealth of procedural skills that are required components of the course and that will likely be targeted in standardized tests. With a limited amount of precious classroom time, how can teachers even hope to accomplish all of these goals? It would be nice if they were able to spend a day, or even a week of their algebra course on helping their students gain a deep understanding of the equals sign, but doing so would prevent getting to the lessons on quadratics at the end of the year. It is unlikely that school administrators and curriculum specialists would be amenable to this solution. Thus, teachers need clever ways of improving conceptual understanding without sacrificing attention to procedural skills.

. . . students with strong conceptual knowledge about a topic are likely to continue to learn more because their prior knowledge makes it easier for them to process and use new information related to that topic.

Fortunately, some such instructional techniques have already been identified by researchers in the domains of cognitive development and cognitive science. One combination which may be especially helpful is the use of worked examples with self-explanation prompts. Worked examples are just what they sound like—examples of problems worked out for students to consider, rather than for them to solve themselves (Sweller & Cooper, 1985). Replacing many of the problems in a practice session with examples of how to solve a problem leads to the same amount of procedural learning in less time (Zhu & Simon, 1987), or increased learning and transfer of knowledge in the same amount of time (Paas, 1992).

When studying worked examples, students should be prompted to explain them. Self-explanation facilitates students in integrating new information with what they already know,

Continued on page 34

and forces the learner to make their new knowledge explicit (Chi, 2000). Typically, students are shown a correct example and asked to explain why the solution is correct. However, explaining a combination of correct and incorrect examples (i.e., explain why a common incorrect strategy is wrong) can be even more beneficial than explaining correct examples alone (Siegler, 2002). Well-designed incorrect examples anticipate common misconceptions that students may hold that would make solving a particular type of problem difficult. For example, students may have a strategy that is perfectly good for some problems (e.g., combine two terms by adding the numbers involved; 4x + 3x is 7x), but misconceptions about the nature of variable versus constant terms lead them to generalize this strategy to other problems where it is not appropriate (e.g., 4x + 3 is not 7x). When students study and explain incorrect examples, they directly confront these faulty concepts and are less likely to acquire or maintain incorrect ways of thinking about problems (Siegler, 2002; Ohlsson, 1996).

If the goal is improving conceptual understanding without harming development of correct procedures, the worked example/self-explanation approach meets that criterion. Many studies have established the benefits for procedural knowledge of worked examples (e.g., Sweller & Cooper, 1985; Zhu & Simon, 1987), and the benefits for conceptual understanding of self-explanation (e.g., Chi, 2000). Further, recent studies have shown that comparison and explanation of multiple correct examples (Rittle-Johnson & Star, 2009) or explanation of a combination of correct and incorrect examples (Booth, Paré-Blagoev, & Koedinger, 2010) can lead to both improved conceptual and procedural knowledge.

Despite their recommendation for instructional use by the U.S. Department of Education (Pashler et al., 2007), research-proven techniques (such as the worked example/self-explanation approach), often fail to find their way into everyday classroom practices or textbooks. This may be because education stakeholders do not believe that they will be useful in real-world classrooms, or perhaps because they see them as incompatible with the setup of typical American classrooms. Greater collaboration between teachers, education researchers, and textbook publishers may be necessary for true change to occur.

In the meantime, the policy makers are right: It is crucial that teachers focus on improving students' conceptual understanding, as misconceptions and impoverished concepts put our students at an alarming disadvantage. But perhaps the best way to help students build a strong concept of math begins with helping teachers build a strong concept of what conceptual knowledge really is. Such a foundation should enable them to devise, implement, and evaluate possible instructional techniques for building strong conceptual knowledge in the classroom.

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Measuring Up: What Teachers Know About Mathematics

by Melissa M. Murphy, Margaret Sullivan, Alexandra L. Chaillou, and Karen E. L. Ross

Early childhood and elementary educators are tasked with the responsibility of promoting student *proficiency* in mathematics (National Mathematics Advisory Panel (NMAP), 2008). *Proficiency* in this context means that "students should understand key concepts, achieve automaticity as appropriate (e.g., with addition and subtraction facts), develop flexible, accurate, and automatic execution of the standard algorithms, and use these competencies to solve problems" (NMAP, 2008, p. 22). By this definition, proficiency requires instilling in students a conceptual understanding of the content (i.e., mathematical ideas) while simultaneously reinforcing procedures (i.e., standard algorithms for problem solving), as elaborated further by Julie Booth in this issue.

The responsibility of promoting student proficiency across a range of learners requires that teachers themselves be proficient in the mathematics content at their grade level, as well as at grade levels above and below their own (Common Core State Standards Initiative, 2010; NMAP, 2008). Teachers benefit from understanding how children think about mathematics (e.g., Carpenter, Fennema, Peterson, Chiang, & Loef, 1989) so that they are prepared to anticipate student misunderstandings, identify common student errors, provide accurate representations of concepts, reason about how students approach problem solving, and identify sources of individual differences in mathematics performance. The purpose of the present article is to discuss teachers' knowledge of mathematics content, identify some of the mathematics content that is challenging for elementary educators and how such challenges might affect student learning, and provide research-based recommendations for how teachers can promote their own mathematics skill development and that of their students.

Teacher Knowledge of Mathematics Content

The NMAP (2008) sets completion of school algebra (i.e., all algebraic material through Algebra II) as the minimum academic goal for all students (p. xvii). To that end, the NMAP charges teachers in kindergarten through 8th grade (K–8) with ensuring student proficiency with the foundational skills of algebra, specifically whole numbers, fractions, and aspects of geometry and measurement central to algebra (e.g., properties of triangles, perimeter, area, volume, and surface area of shapes, determining unknown lengths, angles, and areas). Yet, these foundational skills of algebra and the hierarchical nature of content are areas of mathematics in which many elementary educators lack a strong conceptual understanding (Garet et al., 2010; Hill, Rowan, & Ball, 2005). For example, Ma (1999) found that fewer than half (43%) of teachers from the United States that she sampled correctly completed the problem $1\frac{3}{4} \div \frac{1}{2}$, and none was able to adequately explain the reasons behind his or her calculation (p. 83; see Table 1). Among the misconceptions evidenced by these teachers was confusing division by $\frac{1}{2}$ with division by 2 or multiplication by $\frac{1}{2}$.

Consistent with the results of Ma (1999) and others (e.g., T. Post, Harel, Behr, & Lesh, 1988), we found that more than half of the 23 undergraduates enrolled in a mathematics course for elementary education majors at a women's college in the mid-Atlantic region of the United States (most of whom were elementary education majors) had difficulty with aspects of basic mathematics, including performing mixed operations with fractions, ordering fractions and decimals, and converting from a mixed number to a decimal (Murphy, Sullivan, & Chaillou, 2011). These results were similar to the results of two groups of in-service elementary teachers, which suggests that conceptual difficulties with mathematics content are not necessarily resolved upon completion of teacher training (Murphy et al., 2011).

. . . the magnitude of the effect of teacher knowledge on student achievement is comparable to the effect of student characteristics, such as absence rate, race/ethnicity, gender, and socioeconomic status.

Although content areas (e.g., fractions) are especially challenging for many elementary educators, there is still wide variability in the extent to which educators possess a deep understanding of and comfort with mathematical content (Hill, 2010; T. Post et al., 1988). Such individual differences in mathematics proficiency among educators may, in part, reflect a lack of consistent, clear, and rigorous standards for entry and exit of elementary education programs across the country as well as variability in the relevance, breadth, and depth of coursework required of elementary education teacher candidates (Ma, 1999). For example, the National Council on Teacher Quality (NCTQ, 2008) reports that only 10 out of 77 (13%) schools of education sampled across the United States had programs that provide coursework in content and methods that cover all topics essential for elementary educators (e.g., numbers and operations, algebra, geometry and measurement, and data analysis and probability). Together, these findings support the notion that it is unlikely that elementary

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TABLE 1. Summary of Results for Computation of and Representation for 1¾ ÷ ½ Among Teacher Respondents		
n[a]	Response Examples	Accuracy
	Computation Responses	
9	3½	Correct
2	$\frac{14}{4}$ or $\frac{28}{8}$	Correct algorithm; incomplete answer (answer not reduced)
4	"invert one of the fractions…I'm not sure" (p. 57); "you'd *have to*…change them in sync…cross multiply…You get 28/8?" (p.56)	Unclear about algorithm or procedure; incomplete answer
5	—	Incomplete or inaccurate memory of algorithm, no response given
1	—	Incorrect strategy, no response given
	Representation Responses	
0	how many one halves are in 1¾; use in a representation that produces a possible remainder (e.g., ½ of a mile)	Correct conceptual representation; appropriate pedagogical choice
1	how many one halves are in 1¾; use in a representation that produces an impossible remainder (e.g., ½ of a person)	Correct conceptual representation, poor pedagogical choice
6		Incorrect, no representation provided
16	Confusing division by ½ with division or multiplication by 2	Incorrect representation provided, misconception evident

Note. Table values for computation responses are based upon *Knowing and teaching elementary mathematics: Teachers' understanding of foundation mathematics in China and the United States* (p.58), by L. Ma, 1999, Mahwah, NJ: Erlbaum Associates. Copyright 1999 by Taylor & Francis Group LLC – Books. Adapted with permission.
[a] n = number of respondents with given response; n = 21 teachers attempted the computation, n = 23 teachers attempted the representation.

education candidates will receive the instruction necessary to remediate their own conceptual misunderstandings during the course of their education program (Hill, 2007).

Strong teacher content knowledge of mathematics is necessary for effective teaching, but it is not sufficient (Ball, Hill, & Bass, 2005; Hill et al., 2008). Ball and colleagues introduced the term *mathematics knowledge for teaching* (Ball, 1990) to refer to the mix of content and pedagogical knowledge required to teach mathematics. Such knowledge encompasses a teacher's ability to accurately and thoroughly explain terms and concepts, use appropriate examples and representations during instruction, interpret student responses, present topics in a logical sequence, and evaluate textbook content (Hill et al., 2005, p. 373). As discussed in the following section, teachers who possess robust mathematical knowledge for teaching generally demonstrate a higher quality of mathematics instruction than teachers who lack some or all aspects of this knowledge (Hill, 2010).

Advantages Associated with a Strong Teacher Knowledge of Mathematics

Teacher knowledge of mathematics affects student outcomes in mathematics (e.g., Carpenter et al., 1989; Ma, 1999). For example, as early as first and third grades, the mathematical content knowledge a teacher possesses predicts gains in student mathematics achievement scores on the *TerraNova* achievement test (Hill et al., 2005). This effect of teacher knowledge on student achievement exceeds that of teacher background (e.g., years of experience) and the amount of time spent on mathematics instruction (Hill et al., 2005). Moreover, the magnitude of the effect of teacher knowledge on student achievement is comparable to the effect of student characteristics, such as absence rate, race/ethnicity, gender, and socioeconomic status, on that achievement (Hill et al., 2005).

One way in which a teacher's knowledge affects student outcomes is via the quality of mathematical instruction itself (e.g., Borko et al., 1992; Fennema & Franke, 1992). Frequent mathematical errors (e.g., providing inaccurate or incomplete definitions) and poor mathematical choices (e.g., including lessons with minimal mathematics content) characterize the instruction of teachers with low mathematical knowledge (Hill et al., 2008; Putnam, Heaton, Prawat, & Remillard, 1992). At the same time, when a teacher's knowledge of mathematics is high, instructional quality also tends to be high (Hill et al., 2008). For example, Hill and colleagues (2008) found that scores on a measure of teacher mathematics knowledge for teaching were positively correlated with the frequency of appropriate responses to students (e.g., correctly interpreting student responses, querying student errors to understand the

Continued on page 38

student's thought process), and negatively correlated with the number of linguistic errors (e.g., inappropriate use of mathematical notation, inattention to the mathematical meaning of everyday terms such as table, plane, etc.), and number of nonlinguistic errors (e.g., computational errors, incomplete explanations).

How Might Differences in Teacher Mathematics Knowledge for Teaching and Quality of Mathematics Instruction Affect Student Outcomes?

Differentiated instruction, the practice of tailoring instruction and materials to each child's specific learning needs (see Tomlinson, 1999), requires that a teacher adjust to individual differences in students' prior knowledge and current understanding of mathematics to support students as they acquire greater proficiency. Ways to adapt instruction include, selecting appropriate examples, providing alternative explanations, and recognizing and remediating student misconceptions. To achieve this, the teacher must know not only the mathematics content, but also how students think about mathematics. Ball and colleagues (e.g., Ball, 1990; Hill et al., 2008) refer to this special type of mathematical knowledge of teaching as *knowledge of content and students*.

Educators with strong mathematical knowledge of content and their students' mastery of that content may be more effective than their counterparts with weaker knowledge at identifying and remediating common misconceptions or problems that contribute to poor mathematics performance among students, such as inaccurate or incomplete application of standard algorithms (Hill, Schilling, & Ball, 2004). Indeed, teachers with strong mathematical knowledge are quicker and more accurate in providing corrective feedback, make fewer mathematical errors, and use more precise mathematical language than teachers with weak mathematical knowledge (Hill et al., 2008). However, studies that focus on the relationship between teacher knowledge and student achievement in mathematics have not typically examined the effect of teacher knowledge as a function of student mathematics performance level (i.e., MLD, low achievement, typical achievement). Thus, it is unclear whether teacher mathematical knowledge for teaching differentially affects students with low versus higher mathematics achievement.

What is known is that many children are at risk for poor mathematics performance due to environmental risk factors, such as low income or limited parental education (as reviewed by Jordan & Levine, 2009). Children from such environments may have limited opportunities to develop informal mathematics skills, such as counting and judging quantities, prior to entering kindergarten (as reviewed by Baroody, Bajwa, & Eiland, 2009; Jordan & Levine, 2009). These gaps in informal mathematics knowledge place these children at risk for difficulty with formal mathematics skills, such as calculation, at later grades (Jordan, Kaplan, Olah, & Locuniak, 2006). Among these children, there is a need for teachers with strong mathematical knowledge for teaching who can provide high quality mathematical instruction (Hill, 2010; Jordan & Levine, 2009). Yet, less mathematically knowledgeable teachers tend to be employed in school districts that serve low income communities (Hill, 2010).

Similarly, a growing number of students enter school with limited English proficiency (Fry, 2007; National Center for Educational Statistics (NCES), 2006). These students may be at risk for weak mathematics performance due to environmental risk-factors (as discussed previously, Batalova, Fix, & Murray, 2007) or complications associated with learning mathematics in their non-native language, such as needing to translate information back and forth between languages (Brown, 2005; Lager, 2006; Schleppegrell, 2007). As a result, English language learners may be especially sensitive to the quality of mathematics instruction, particularly as it relates to the teacher's frequency of linguistic errors (e.g., inappropriate use of mathematical notation, inattention to the mathematical meaning of everyday terms such as table, plane, etc). Moreover, many in-service mathematics teachers lack appropriate training for teaching English language learners (NCES, 2002) and report feeling less competent working with these students than with their English-proficient peers (Ross, 2011). However, additional studies are needed to assess the relationship between teacher mathematical knowledge for teaching and mathematics outcomes of English language learners.

> ***. . . just like their students, teachers are likely to exhibit a profile of relative strengths and weaknesses (in mathematics).***

When examining the quality of mathematics instruction and its effects on student outcomes, it is important to recognize that teachers may not be universally strong or poor at all mathematic domains—rather, just like their students, teachers are likely to exhibit a profile of relative strengths and weaknesses. Consequently, teaching quality may vary within or across lessons or units, with a greater range of problem types and a higher degree of student-directed discussion of problem solving strategies associated with stronger content knowledge (e.g., Fennema & Franke, 1992, pp. 149-150). As such, it is the consistency of high quality mathematics instruction that distinguishes teachers with strong knowledge of mathematics for teaching from those with less well-developed knowledge (Hill et al., 2008). Although all educators can benefit from ongoing professional development, it is this latter group of teachers for whom strengthening mathematical knowledge for teaching is most urgent.

Promoting Mathematical Skill Development: Recommendations for Teachers

Best intentions and beliefs about effective practices cannot overcome a lack of personal knowledge of mathematics (Ma, 1999). The following are recommendations for teachers as to how they can improve or refine their own mathematics knowledge for teaching.

Familiarize yourself with the mathematics taught at grade levels above and below your grade. Given the effect of teacher knowledge of mathematics on student achievement, the NMAP (2008) recommends that teachers "know in detail the mathematical content they are responsible for teaching and its connections to other important mathematics, both prior to and beyond the level they are assigned to teach" (p. 37). In this way, teachers can help students to see how mathematical ideas are related across grade level (e.g., connecting operations with fractions to those used with whole numbers), which is one of the Standards for School Mathematics set forth by the National Council of Teachers of Mathematics (NCTM, National Council of Teachers of Mathematics, 2000).

Build your mathematics knowledge base. Most, if not all, teachers engage in some type of professional development (NCES, 2005). Yet, research evidence supporting the efficacy and quality of specific approaches to professional development and their effects on student learning are lacking (as reviewed by Hill, 2007; NMAP, 2008). Among the programs that have the greatest effects on teachers' self-reported learning outcomes and subsequent teaching practice are those that focus on helping teachers learn subject-specific content (e.g. rational numbers, geometry, measurement), provide opportunities for teachers to actively engage with the content (e.g., reviewing student work, analyzing instructional examples), and are of long duration (e.g., high number of contact hours over prolonged period; Garet, Porter, Desimone, Birman, & Yoon, 2001).

Another way to build mathematics knowledge for teaching is through "lesson study" (G. Post & Varoz, 2008) or "teaching research groups" (Ma, 1999, p. 136). These regular meeting times for teachers provide a forum in which teachers work collaboratively to enhance the quality of mathematics instruction. Specifically, weekly or monthly meetings provide a formal opportunity for teachers to discuss mathematics across the curriculum, get feedback or guidance on approaches to teaching specific content, reflect on specific classroom experiences, review videotapes of mathematics lessons, or share student-generated strategies (as reviewed by G. Post & Varoz, 2008). Although there is limited research-based evidence supporting the efficacy of such groups on instruction and subsequent student outcomes within the United States, these practices are commonly used in other countries, such as Japan and China (Ma, 1999, p. 136; G. Post & Varoz, 2008), where student achievement in mathematics exceeds that of the United States (Organization for Co-operation and Economic Development, 2010).

At an individual level, techniques, such as keeping a mathematics journal or videotaping mathematics lessons can be helpful (see Jacobs, Ambrose, Clement, & Brown, 2006; Wolhunter, Breyfogle, & McDuffie, 2010). For example, journaling can be used to make notes at the end of class (or day) on specific mathematics lessons (e.g., suggestions for improvements, student comments) or to build a repertoire of ideas related to instruction (e.g., useful metaphors or examples). Videotaping lessons provides the opportunity to look for visual clues of what students are thinking and understanding, appraise and reconsider responses to queries, and reflect on how the lesson could be taught better (Jacobs et al., 2006).

Seek mathematics-specific support. National mathematics organizations, in particular the National Council of Teachers of Mathematics (NCTM; www.nctm.org), issue publications specifically designed as professional resources for teachers. Among the publications are grade-level journals that provide specific instructional strategies, case studies, and topics for lesson studies. These publications provide an opportunity for teachers to connect with professionals in the field of mathematics instruction outside of their local school or district.

An additional source of mathematics-specific support available in some schools is the mathematics specialist or coach. Approximately 23% of schools are employing mathematics coaches and about 35% are employing mathematics specialists (NCES, 2009). The role of these "teacher-leaders" as defined by the NCTM (2000) is "assisting teachers in building their mathematical and pedagogical knowledge" (p. 375). Additional research-based evidence is needed to support the efficacy of specialist/coaches on the quality of classroom instruction (Hill, 2010); however, many mathematics specialist/coaches are valuable resources for building content knowledge and improving the quality of mathematics instruction.

Conclusion

The responsibility of promoting students' mathematics proficiency requires that teachers possess the content and pedagogical knowledge necessary for teaching mathematics. As both purveyors of knowledge and models of lifelong learning, it is critical that novice and experienced teachers work continually to strengthen their mathematics knowledge for teaching by seeking out or creating formal (e.g., ongoing professional development) and less formal (e.g., lesson studies) opportunities to engage in teaching and learning mathematics.

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Continued on page 40

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Motivating Students Who Struggle with Mathematics: An Application of Psychological Principles

by Laurie B. Hanich

Much research on children who struggle with mathematics, including students with mathematics learning disability (MLD), is focused on cognitive skills required for success on mathematics tasks, such as number sense or computational fluency (as summarized by Michèle Mazzocco, this issue). Identifying these skills is essential to targeting educational supports. Yet a variety of noncognitive factors, such as affect and motivation, also play a critical role in children's learning of mathematics (Royer & Walles, 2007). For example, some children who struggle with math demonstrate maladaptive patterns of motivational behavior, such as disengaging in mathematics instruction or demonstrating self-handicapping avoidance behaviors. While these behaviors often serve as a defense mechanism that protects one's self image (Covington, 1992), a lack of engagement during mathematics instruction can affect a child's mathematics learning. Although engagement and motivation can influence math learning for all children, fostering adaptive motivational behaviors is essential for children who experience difficulty with mathematics.

Factors other than inherent ability influence how students approach classroom tasks, the types of strategies students use, and how students respond to academic successes or failures.

Although few educators would argue with the need for students to be motivated, many do not fully understand the complexity of achievement motivation (Anderman & Anderman, 2010). Factors other than inherent ability influence how students approach classroom tasks, the types of strategies students use, and how students respond to academic successes or failures. Teachers can influence students' achievement motivation and have the potential to help or hinder children's mathematics learning. The intent of this article is to provide a brief introduction to the principles from the achievement motivation literature in order to assist educators with creating positive environments that facilitate learning by motivating students. Below, I first describe maladaptive motivational profiles that characterize some students who struggle with math and explore potential consequences of such behaviors. Then, I identify and describe five principles of achievement motivation that educators can apply to motivate students who struggle in math. These principles extend to all learners and cut across content areas; however, there may be particular value in their application to children who struggle with math.

Maladaptive Motivational Profiles That May Characterize Children Who Struggle with Math

Madison is a third grade student in Mrs. Burkhardt's classroom. For the last 15 minutes she has been doodling on the workbook page in front of her. Mrs. Burkhardt walks by and encourages her to try one of the story problems that she is supposed to be working on. "I can't," claims Madison, "I don't understand it." Mrs. Burkhardt goes through the problem step by step, checking Madison's understanding along the way. Together, they solve the problem correctly. "Try the next one on your own," says Mrs. Burkhardt. "I don't know how," replies Madison. "But we just did one like this," a frustrated Mrs. Burkhardt responds. "You helped me. I can't do it on my own," Madison insists. "But you knew which steps to use when we solved it together," Mrs. Burkhardt argues. "I guessed," replies Madison. Mrs. Burkhardt moves on to assist another student and Madison returns to her doodling.

There are several possible cognitive and behavioral explanations for Madison's actions, including her perception that she lacks control over her mathematics success. It is possible that her self-perception is accurate—some children are aware of real limitations that prevent their success on a given task; or hers could be a false sense of futility resulting from experiencing repeated failure. This *learned helplessness* leads to reliance on task-avoidant strategies such as disengagement. If students have low expectations for success on a task, they may be more likely to give up easily, attribute their failure to their lack of ability, and attribute their success to external factors, such as luck or ease of the task.

Recognizing task-avoidant strategies in children who struggle with math can help teachers respond appropriately to these maladaptive behaviors. However, teachers can gain additional insights into their students' problems with math if they consider both cognitive and motivational variables that affect their students' academic performance and contribute to task-avoidant behaviors. For example, children with weak executive control or poor working memory may be less capable than their peers in handling some of the task demands associated with mathematics, as described by Daniel Berch (this issue). As Dr. Berch suggests, it is possible to diminish working memory demands to enhance mathematics learning and performance of children with low working memory capacities. Still, it is important to recognize that repeated experiences of failure can interact with cognitive motivational variables, such as children's belief systems. Cognitive competence and motivation are dynamically related (Chapman, 1988; Royer & Walles, 2007), and

Continued on page 42

children's belief systems in turn affect adaptive behaviors for mathematics learning. Ashcraft and Kirk (2001) found that children who are highly anxious about mathematics showed longer response times and less accuracy on two-column addition problems that involved borrowing than children with lower levels of math anxiety. Although it is difficult to determine whether low mathematics performance precedes high mathematics anxiety level, or vice versa, regardless of their causal relationship, it is imperative to examine motivational behaviors.

Breaking the cycle of learned helpless behaviors is difficult, but important. If young children develop this pattern of behavior early in their academic career, their beliefs often are reinforced over time and compounded by cognitive deficits associated with MLD. Teachers should help students understand the effects of their attributions, especially when they have failed at a task. For example, student failure on tasks might be related to the selection of problem-solving strategies rather than an innate ability in mathematics. This type of message is particularly important for children with MLD, because they often use immature strategies relative to their typically achieving peers (Geary, 1990; Ostad, 1997, 1998; Russell & Ginsburg, 1984).

A relevant contributing factor to learned helplessness could be the social interactions with classroom teachers who unintentionally communicate low expectations to students regarding their mathematical abilities. Teachers who believe that mathematics is difficult and that some children lack the cognitive competencies to be successful in mathematics may not fully understand the impact of such beliefs on their students' achievement-related perceptions. Beliefs held by teachers about their students' mathematical abilities may contribute to the development of a self-fulfilling prophecy. For example, Bielock, Gundeson, Ramirez, and Levine (2010) recently found that female teachers' math anxiety affected first- and second-grade girls' mathematics achievement gains. At the end of the school year, girls with highly math-anxious teachers were more likely than boys and than girls with less math-anxious teachers to endorse the stereotype that boys are good at math and girls are good at reading. Also, the mathematics achievement of these girls was significantly lower than that of boys and of girls who did not endorse this stereotype. This suggests that teacher behavior *can* and *does* play a major role in student learning.

> *John is a fifth-grade student in Mr. Marshall's classroom. Although John is a skilled reader, he has struggled in math throughout most of elementary school. He has a test on fractions in his math class tomorrow. He knows that he needs to study for his test, but instead he spends the entire evening playing video games with his brother. When he fails the test the next day, he blames his failure on his time spent playing video games. A similar pattern of behavior has happened several times earlier this year. Mr. Marshall has become increasingly frustrated by John's repeated lack of effort.*

Like Madison, John is engaging in another form avoidance behavior, self-handicapping. Unlike learned helplessness, self-handicapping is a proactive strategy that students use to influence others' beliefs about their ability (Urdan, Ryan, Anderman, & Gheen, 2002). In these instances children's likelihood for success is usually diminished because they have engaged in behaviors that are counterproductive to the task at hand. Students who self-handicap often do so because they doubt their ability to perform adequately on academic tasks. To avoid the implication that he lacks ability in math, John blames his failure on something within his control, studying for the test. This allows him to maintain positive perceptions of his abilities, despite his concerns about his competence. It is unknown to what extent self-handicapping occurs in children with MLD.

If students have low expectations for success on a task, they may be more likely to give up easily, attribute their failure to their lack of ability, and attribute their success to external factors, such as luck or ease of the task.

What can teachers do to assist students like Madison and John? In addition to their limited proficiencies in certain areas of mathematics, students who engage in unproductive motivational behaviors are likely to augment their problems in math. Explicit instruction with students who struggle with math has consistently shown positive effects on performance with word problems and computation (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2009; National Mathematics Advisory Panel, 2008; Powell, Fuchs, & Fuchs, this issue). However, given the interplay of cognition and motivation, teachers can supplement explicit strategy instruction with application of the following psychological principles to foster adaptive motivational behaviors in students who struggle with math.

Principles of Achievement Motivation

1) **Promote mastery goals and minimize performance goals in the classroom.** *Achievement goal orientation* refers to the way that students approach, respond to, and engage in achievement-related activities (Ames, 1992). Researchers have identified two goal orientations: *mastery goals* and *performance goals*. Mastery goals are characterized by an emphasis on mastering content, increasing knowledge, and developing competence. Behaviors associated with mastery goals include risk taking, utilization of sophisticated problem-solving strategies, adaptive help-seeking, and perseverance. In contrast, performance goals are characterized by an emphasis on demonstrating competence and avoiding situations that have potential to reveal incompetence. Behaviors associated with performance goals include a

focus on grades and social comparison, selection of easy tasks that are likely to guarantee success, and utilization of shallow problem-solving strategies.

Students' achievement goals can be influenced by the way that teachers structure the classroom (Ames & Archer, 1988). Instructional tasks, feedback and recognition practices, and assessment materials have an impact on children's goal orientations, which in turn affect achievement-related behaviors. For example, Turner and her colleagues (Turner et al., 2002; Friedel, Cortina, Turner, & Midgley, 2007) found a relationship between children's perceived goal structures of the classroom and their use of avoidance behaviors and coping strategies. Specifically, in classrooms where a performance goal orientation was salient (e.g., high demand for correct answers, but with little explanation or support for arriving at such), students were more likely to engage in avoidance behaviors or other unproductive behaviors. In classrooms where children perceived a mastery goal orientation (e.g., emphasis on understanding procedures and concepts as a means to arrive at correct answers), children were less likely to engage in avoidance behaviors. These findings illustrate the importance of creating classrooms that minimize performance goals over mastery goals without eliminating the importance of both procedural and conceptual learning (see Julie Booth's article, this issue).

So how do teachers promote mastery goals over performance goals in the classroom? One way is to emphasize the incremental nature of developing mathematical competence. If teachers send the message that success in mathematics is the result of self-regulated strategy use and persistence on challenging tasks, rather than the result of innate ability, children are more likely to attempt problems outside of their comfort zone and to respond adaptively following a failure situation. On the other hand, classrooms that emphasize public recognition of ability, promote feedback based on ability rather than effort, and that discourage educational risk-taking for fear of making errors, are likely to reinforce task-avoidant behaviors. By creating a classroom where students can focus on the process of learning, rather than the product of performance, students are more likely to demonstrate positive motivational behaviors, such as engagement and resiliency.

2) **Minimize social comparison and rewards based on performance.** Many educators are partial to the use of rewards as a motivating strategy, and research has identified several ways that rewards can be used effectively (see Anderman & Anderman, 2010, for a review). However, when used in ways that are inconsistent with research-based guidelines, rewards have the potential to foster task-avoidant behaviors (Deci, Koestner, & Ryan, 2001). This may be particularly true for children who struggle with math or have MLD. For example, in some elementary classrooms it is a common practice for students to complete "fast facts" worksheets, where students have to retrieve as many number combinations as possible in a two-minute time period. At the end of this activity, the child with the highest score is often identified as the winner, and winners often receive a prize or have their name posted in the class as a form of recognition. Although practicing number combinations has important benefits (see Sarah Powell and her colleague's article, this issue), from a motivational perspective, there are several problems with the reward structures used in this example. First, if only one child is identified as the winner, it might create feelings of resentment among classmates. Second, it is likely that the same child (or one of a small subset of students), usually a high achiever, wins this activity multiple times. Third, the likelihood of a child with mathematics difficulties winning this activity is slim because deficits in retrieval of arithmetic combinations are a defining characteristic of these children (Hanich, Jordan, Kaplan, & Dick, 2001). Finally, when a reward is available for only a select group of children rather than the entire class, there is potential to foster avoidance behaviors among those who perceive their chances of winning as low (Deci et al., 2001). While most students are likely to make performance gains on math facts tasks over time, their gains are unlikely to elevate them to the position of "the winner."

In addition to practices that utilize rewards, those that foster social comparison can be equally as perilous for children who struggle with math. Consider that many teachers have classroom charts that identify the level of the multiplication table facts that students have mastered. While motivating to students who are at the top of the chart, students who struggle with math might find this practice embarrassing because it draws attention to their low performance. Many children would prefer to be seen as "bad" rather than "dumb," so it is not uncommon for children who struggle with math to engage in task-avoidant behaviors when social comparison is prominent.

Although automaticity with number combinations is an important skill for developing mathematical competence, the types of reward-based practices exemplified above have the potential to foster negative achievement-related behaviors. There are other ways that this essential skill can be monitored without fostering social comparison among students. Teachers can still administer "fast fact" sheets to monitor student skill level, but base recognition on individual student improvement and progress. Feedback about performance should be kept private and children encouraged to focus on self-set standards rather than normative standards. Finally, effort and persistence should be recognized and appropriately reinforced.

3) **Help students understand the effect of negative attributions.** *Attributions* are causal explanations that children make to explain their academic successes or failures (Weiner, 1986). Consider the student who has experienced repeated failures in mathematics. When probed about the causes of such failures, the child states, "I'm not good at math. No one in my family is good at math." This attribution identifies inherent ability as the underlying cause of mathematical performance. If children believe that their mathematical failures are due to lack of ability, something

Continued on page 44

outside of their control, they could be less likely to engage in similar tasks in the future since they believe it is likely that the outcome will be the same. However, if children believe that their mathematical failures are due to limited effort or poor strategy selection, things that are within their control, the likelihood that they engage in future tasks is high if the outcome has the potential to change.

To help students replace dysfunctional attributions with facilitative ones, teachers should foster attributions that emphasize effort, strategy use, and persistence, over ability (National Mathematics Advisory Panel, 2008). All of these attributions promote an *internal locus of control*, in which children perceive themselves as active agents of their learning, rather than passive recipients of external forces (Pashler, McDaniel, Rohrer, & Bjork, 2008). Additionally, teachers need to be aware of the unintended mixed messages they send to students, which affect the types of attributions that students make about themselves. For example, a teacher might not question making references to how "smart" children are. However, this attribution is likely to focus children on *ability* as the cause for their success, rather than *effort*. While this may not seem counterproductive to children's developing self-concepts, imagine the inverse attribution in a failure situation. Being "dumb" not only has a negative impact on a student's academic self-concept, but ability attributions tend to be relatively stable across time. Thus, stable, external attributions of failure often are outside of children's control, which has consequences for future behaviors. Finally, teachers need to continually battle the erroneous idea that mathematical competence is largely a matter of inherent ability, not effort and convey this message to students in their own instructional practices.

4) **Teach students appropriate self-regulated learning strategies.** Self-regulation strategies include self-instruction, self-questioning, self-monitoring, and self-assessment. These metacognitive functions are critical for children's academic success. Research has indicated that children with MLD are less accurate at evaluating correct and incorrect solutions, and are less accurate at predicting which problems they can solve correctly, than children without MLD (Garrett, Mazzocco, & Baker, 2006). While intervention studies that use self-regulation strategy instruction to improve mathematics performance have not been conducted specifically on the MLD population, several researchers have reported positive effects in populations of students with general learning disabilities (LD). Jitendra and her colleagues (Jitendra, Griffin, McGoesy, Gardill, Bhat, & Riley, 1998; Jitendra & Hoff, 1996) reported an increase in mathematical problem solving in elementary school students with LD using schema based instruction, which teaches students to categorize word problems into different types and then implement the appropriate strategy. Similarly, Sarah Powell and her colleagues (this issue) demonstrated that promoting strategy use improved number combination skills in third graders with math difficulties, and Montague (Montague, 1992; Montague, Applegate, & Marquard, 1993) found a significant improvement in problem-solving performance among older middle school students using *Solve It!* Common features across these interventions include an emphasis on student monitored problem-solving performances by using a variety of self-regulated learning strategies, which are first modeled by the teacher and then taught to children. The goal is that children will acquire and internalize strategies and apply them independently to guide mathematical performance.

Self-regulated learning strategies are important because in addition to guiding children's performance, they also have an impact on children's self-efficacy judgments. *Self-efficacy* refers to an individual's domain specific judgments about their capability to perform a task (Bandura, 1986; Pajares, 1996). As children experience success in mathematics, their self-efficacy is likely to increase. Higher self-efficacy is likely to contribute to a host of adaptive motivational behaviors including engagement, perseverance, and appropriate strategy use (Pajares, 1996). To assist students in developing self-regulated learning strategies, teachers need to model the appropriate strategies for students in addition to developing instructional activities that promote independent and strategic learning. Once children are provided with a variety of strategies that can be utilized on specific tasks, teachers can provide specific feedback about how and when to apply strategies. Finally, teachers can provide specific feedback about the outcome of the strategy application, noting what contributed to children's success or failure on the task.

5) **Model your own value of mathematics.** Despite the fact that some teachers do not enjoy teaching mathematics or their enjoyment of teaching mathematics is not consistent (Stipek, Givvin, Salmon, & MacGyvers, 2001), it is important that teachers model their own value of mathematics. Communicating the value of learning mathematics can help students internalize their own positive beliefs about mathematics. As mentioned earlier in this article, Beilock et al. (2010) found teachers' math anxiety had consequences for girls' math achievement by influencing the girls' beliefs about who is good at math. If students think they are capable of succeeding on a task and perceive the task as having value, they are more likely to attempt it (Wigfield & Eccles, 2000). However, if children believe that they are not capable of success, or if they see little value in the task, they may avoid it.

Teachers' knowledge of mathematics may affect their own enthusiasm for the discipline, which can improve the quality of their mathematics instruction (as discussed by Dr. Murphy and her colleagues, this issue). To help students also see the value of learning mathematics, teachers are encouraged to reflect on the design and implementation of instructional activities in mathematics and to monitor their communication about the importance and relevance of mathematics. Questions for reflection include the following:

- How interesting or enjoyable is the task at hand for students to learn and for me to teach?
- How will success on this task contribute to future mathematics learning among my students?
- How do I communicate messages about the role of effort and ability in learning?
- How do I communicate my own beliefs about the importance and value of mathematics to students?

Conclusion

The principles identified above are supported by educational and psychological research within the field of achievement motivation. All of these principles can be extended to a larger student population; however, for the purposes of this article I have targeted principles that are most applicable for teaching children who struggle with math, including children with MLD. Teachers are cautioned that these principles are not infallible and there are other important variables, such as developmental differences, group and individual differences, and differences in the learning contexts that shape and influence children's motivation, as reported in this issue. Because of the unique characteristics of children with MLD (e.g., cognitive deficits, immature strategy use, poor evaluation skills, etc.), teachers will want to reflect on the application of these motivational principles, adapting as necessary to students' individual characteristics and unique classroom environments. It is my hope that research on children who struggle with math or have MLD will continue to incorporate cognitive-motivational factors into systematic investigations of mathematics achievement that will provide educators strategies and recommendations to improve classroom instruction of mathematics.

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Number Matters

by Michèle M. M. Mazzocco

Why do some students have difficulty with mathematics, and how often do these difficulties indicate a mathematical learning disability (MLD; a term used here as synonymous with dyscalculia)? How early can we identify which children are at risk for poor mathematics outcomes? These are the questions that motivated a research program through which my colleagues and I studied early cognitive predictors of mathematics achievement. During a conference presentation on some of our findings, members of the audience challenged our focus on cognition, arguing in favor of studying influences that they emphasized could be controlled, namely curriculum and instruction. In partial support of these protests, this special issue of *Perspectives on Language and Literacy* illustrates many possible sources of mathematics difficulties, including external sources that are not attributable solely to characteristics of the struggling learners themselves. Instructional content, instructional quality, and teachers' knowledge of content and learning, are some of the external factors addressed by the authors of the preceding articles in this issue. An awareness of these factors reminds us to evaluate children *in context*—as learners in classrooms or other learning environments—when determining possible sources of their learning difficulties.

However, the importance of external factors does not diminish the significant impact that child characteristics, including cognitive abilities, also have on mathematical learning. As the other articles in this issue reveal, cognitive skills may interact with external influences, but they may also be more directly tied to mathematical difficulties even when learning environments themselves are optimal. Several cognitive characteristics are relevant to mathematical learning. In this article, I focus on abilities that specifically concern understanding and using numbers to describe why children's number skills matter to mathematics, and why they should matter to educators.

Children's Sense of Number

Although *number* alone does not define mathematics, understanding number is essential to the mathematics we learn in school. Why, then, is the role of number often minimized or ignored when seeking explanations for children's mathematical difficulties? Perhaps skills related to number appear insignificant amidst the many alternative explanations for mathematics difficulties—as aforementioned. Moreover, our ability to understand number may be perceived as too simplistic to affect performance on complex mathematics. Regardless of whether number skills are considered easy or challenging, many persons believe that a knack for numbers cannot be changed simply because of its inherent nature. Of course, each of these premises is false. A sense of number *does* play a significant role in mathematical learning, in addition to other factors. Allegedly "simple" number skills are comprised of complex functions, as I describe below. Even if ability levels for some number skills are difficult to improve (if that proves to be the case), an awareness of the cognitive skills that mediate mathematics learning and performance can guide instructional decisions and strategies. Finally, inherent skills *are* subject to change, despite the enduring myth that they are immutable (Sternberg & Grigorenko, 1999). Misconceptions that portray number skills as insignificant or unchangeable prevent educators and parents from recognizing just how much number matters to a child's mathematical learning.

Misconceptions that portray number skills as insignificant or unchangeable prevent educators and parents from recognizing just how much number matters to a child's mathematical learning.

A child's awareness and comprehension of numbers is often referred to as "number sense." Individual definitions of number sense vary in their specificity or breadth of focus (see Berch, 2005 for a review), as is also true for definitions of mathematical learning *disability* (MLD) or math *difficulties* (MD) (as briefly discussed in the "Theme Editor's Summary"). Dehaene (1997) refers to number sense as, "a direct intuition of what numbers mean," citing Dantzig's 1954 introduction of the term to describe a "faculty" that permits someone to recognize that the quantity of a collection has changed without direct knowledge of that change. Dantzig's description pertains to the essence of quantity *per se,* rather than to symbols that represent quantities via spoken or written words (e.g., *three* and *eight*) or numerals (e.g., *3* or *8*). Some cognitive scientists are seeking to further delineate components of number sense, to identify how these components change as children develop, if these components are prone to individual differences, and if they are essential for achievement in mathematics.

Components of Number Sense

Several components of number sense have been described to date, although many have not been studied relative to mathematical difficulties. Here I review only a few components, to illustrate the range of skills involved in number sense.

Number skills can be differentiated according to whether quantities are represented nonsymbolically (sets of items, as in Dantzig's description) or symbolically (e.g., numerals). Nonsymbolic tasks may involve, for example, judging which of two arrays of items is more numerous without counting. For instance, you can readily judge that the paragraph you are now reading has more words than the immediately preceding paragraph, without knowing how many words appear in either. Likewise, if asked to judge which of two words has more letters, you can quite readily determine that there are more letters in

Continued on page 48

the word *mathematics* than in the word *science*. Moreover, your judgment is likely to remain accurate even if the words science and mathematics both cover comparable lengths of horizontal space within text, and even when the "smaller" word (*science*) is physically larger. This ability to nonverbally perceive and represent quantity is observed in infants and preschoolers, and even non-human animals. It is described as informal, or unlearned, and appears to be innate. It generalizes to non-visual stimuli, so it is not simply an artefact of visual perception. (See Feigenson, Dehaene, & Spelke, 2004, for a review).

The early emergence of this type of number sense suggests (inaccurately) that all children will have an adequate number sense by the time they begin kindergarten. Although this may be true for many children, basic nonsymbolic skills improve with development before and well beyond kindergarten (e.g., Halberda & Feigenson, 2008), with performance accuracy levels varying even among high school students. But do these skills matter? The evidence suggests that they do. Performance on these basic informal skills predicts primary and high school mathematics achievement (e.g., de Smedt, Verschaffel, & Ghesquiere, 2009; Halberda, Mazzocco, & Feigenson, 2008). However, "number sense" predictors also include formal symbolic skills that are the target of early instruction.

Symbolic skills are learned, and there is much to learn about them. For instance, we learn that symbols are associated with their nonsymbolic referents, such that the symbols *5* and *five* refer to a specific quantity (e.g., $\boxed{\bullet\bullet\bullet\bullet\bullet}$ but not $\boxed{\bullet\bullet\bullet}$ or $\boxed{\bullet\bullet\bullet\bullet\bullet\bullet\bullet}$). Although this formal association is a focus of school mathematics, it may build upon the informal sense of number that children establish prior to schooling through their daily interaction with the environment. As straightforward as this association may appear to adults, there is a developmental time course when these associations form and become automatic. Within age groups, there are individual differences in both accuracy and automaticity. This means, for instance, that a child who has difficulty quickly identifying that there are five dots in this box $\boxed{\bullet\bullet\bullet\bullet\bullet}$ may have difficulty in one (or more) other skills, including accurately representing the quantity of five dots, forming an accurate association between the quantity and its corresponding symbol (*five* or *5*), or retrieving an association that is accurately established.

When evaluated in children (or adults), these and other types of number sense skills are often measured as automatic skills, typically too subtle to be observed casually. However, recent findings suggest that individual differences in math performance among typically achieving students (those who do not have MLD, but who nonetheless present with a wide range of achievement levels) are related to intentional processing of numerals (e.g., Bugden & Ansari, 2011). This introduces another level of complexity in differentiating effortful or deliberate versus automatic skills. The more deliberate and more effortful that a task becomes, the more it is potentially subject to other cognitive skills.

In other words, the effects of number are not absolute; they interact with effects of other cognitive characteristics. For example, Jordan (2007) has shown that children who have difficulties in both mathematics and reading have weaker performance on tests of approximate calculations, such as determining if the solution to 9 + 7 is closer to 20 or 40, relative to children with mathematical difficulties only. (Both groups perform more poorly on this task than their typically achieving peers.) However, these two groups of children with MD have comparable levels of performance on measures of exact mathematics calculation, such as determining the precise solution to 9 + 7, both performing more poorly than their typically achieving peers. Thus, number sense difficulties in children with MD vary depending on children's reading abilities (See Jordan, 2007, for a review.)

Why do these number skills matter? They exemplify that at least some components of number sense seem very intuitive, and that children differ in the ease with which they process these aspects of number. A sense of number at this level does not emerge simply from being told the properties of number. However, the term *number sense* is sometimes used to refer to skills that are explicitly taught, as indicated in the following excerpt from the National Mathematics Advisory Panel (NMAP) final report (2008, p. 27):

> A more advanced type of number sense that children must acquire through formal instruction requires a principled understanding of place value, of how whole numbers can be composed and decomposed, and of the meaning of the basic arithmetic operations of addition, subtraction, multiplication, and division. It also requires understanding the commutative, associative, and distributive properties and knowing how to apply these principles to solve problems.

Unlike the previously described basic number sense skills, the skills referred to in this definition reflect cognitive characteristics that are not necessarily limited to numerical tasks alone. Rather, they include general cognitive skills (e.g., working memory) that affect learning across domains, including reading comprehension and writing. One can speculate that both basic and general skills influence mathematics performance on tasks such as number combinations and algebra, that both have a role in procedural and conceptual learning, that both affect student engagement, and that knowledge of these diverse skills may affect how teachers adapt instruction for individual students across all achievement levels.

It can be difficult to differentiate numerically specific versus general cognitive contributions to a child's mathematical difficulties. To solve problems such as 7+14 or $\sqrt[3]{27}$, a child must know and recall what the number symbols mean, what computational procedures to use, and how to execute the procedures correctly. Knowing why these procedures are executed offers additional benefits, but this computational knowledge and recall can occur with varying degrees of understanding or efficiency. Once solutions are learned, stored facts may be retrieved from memory or via other routes (e.g., LeFevre et al., 1996), or backup strategies may be used (as discussed by Sara Powell and her colleagues in this issue). A solution can be calculated by counting up (see Powell, this issue) from either 7 or 14; the counting may occur with or without using fingers or tally marks. Alternatively, a child may decompose 14 into 7 and 7, which when combined with the first addend (another *7*), is solved as $3\times7=21$. When decomposing number, a child may rely on procedural or conceptual knowledge and on automatic or more effortful processes. Cognitive processes might vary if the tasks require finding an approximate versus exact solution or if the tasks occur under timed versus untimed conditions. The child's processes may even vary depending on whether the problem is presented horizontally or vertically. Moreover, problem solving strategy choices vary with age and within individuals at a given point in time. Sometimes, individuals are not aware of the skills they are using. This adds to the existing challenges in identifying which specific skills underlie mathematical difficulties for a given child.

Among the many possible skills that support or hinder mathematical learning, some of these skills, including number sense related skills, can be improved during early childhood. Research in this area is ongoing. Much of the research demonstrating success of number sense interventions has been conducted with children from low income households, in which case poor number skills may be linked to low numeracy environments. In these instances, even small improvements in a child's environment can make a difference. Siegler and his colleagues have shown that preschoolers' representation of numbers on a mental number line improves after playing a numerical board game just a few times per week. The preschoolers' performance also improves on number recognition and numerical comparison tasks (Siegler, 2009). Not surprisingly, the amount and type of numerically oriented activities in the child's home is related to children's math and number skills at the onset of schooling (Blevins-Knabe, 2008).

Why do these interventions matter? Because number skills support learning school mathematics, and because number difficulties will not simply disappear with time. Indeed, poor mathematics knowledge is present in adults, including teachers (see Murphy et al., this issue). Poor number sense in adults hinders skills we rely on for occupational, leisure, and daily activities, ranging from engineering to carpentry, sports to card playing, healthcare risk assessments to financial decision making (see McCloskey, 2007, for a review). Thus, number interventions matter, because, throughout life, number matters.

Number Sense Skills in Children with MLD

In view of the many cognitive skills involved in mathematics, it is no wonder that no single core deficit has been identified for mathematics learning disabilities (MLD), and it is possible that no single best predictor of mathematical difficulties (MD) will ever emerge. Until core deficits are identified, achievement levels will probably continue to be used in research as proxies of MLD or MD. In earlier research, MLD and MD were rarely differentiated from each other. Whether the term MD or MLD was used, classification was often based on relatively high math achievement cutoff scores (e.g., at or above the 25th percentile). This practice resulted in unintentionally combining groups of individuals whose difficulties stem from different causes. We and others have since found important differences between children who meet stricter criteria for MLD (e.g., performing at or below the 10th percentile) versus those who meet more lenient criteria (e.g., performing within the 11th to 25th percentile (e.g., Murphy, Mazzocco, Hanich, & Early, 2007). Many now refer to these groups as students with MLD or Low Achievement in mathematics, respectively (e.g., Geary et al., 2007). In studies, each of these groups is usually also compared to a group of students who have age- and grade-appropriate mathematics achievement. In this way, we move closer to differentiating children with MLD from other children with other sources of math difficulty.

Number sense skills are among the many cognitive variables that differentiate children with MLD from those with low math achievement. For instance, children with MLD have significantly weaker performance than their typically achieving peers on measures of basic, automatic nonsymbolic number sense (e.g., selecting which of two sets of items is more numerous, without counting) and on tasks that measure associations between quantities and verbal labels. Importantly, children with MLD perform more poorly on these intuitive tasks than children with low math achievement. In contrast, children with low math achievement are indistinguishable from their peers on these tasks (Mazzocco, Feigenson, & Halberda, 2011), which suggests that their math difficulties emerge for other reasons—including (potentially) other cognitive skills. Still, this finding does not mean that children with low math achievement necessarily have intact number sense overall, because, as reviewed thus far, there are many components of number sense.

Summary

Mathematics relies in part on a sense of number, or quantity. If we take these basic skills for granted, we may miss the very foundation of some children's mathematical difficulties. Why shouldn't there be individual differences in how readily we perceive and process number, or how readily we form associations between quantities and formal symbols? Decades of research on dyslexia has shown us that there are individual differences in how readily children process the building blocks of language. The same may well be true for children with dyscalculia, or MLD.

Difficulties in the most basic, inherent aspects of number sense are not the cause of all mathematical *difficulties*. As this special issue of *Perspectives on Language and Literacy* demonstrates, mathematical difficulties may be attributable to other child characteristics associated with cognition (such as working memory), psychosocial function (e.g., mathematics anxiety, self handicapping behaviors, etc.), or behavior (e.g., engagement); it may result from compromised learning opportunities, or some combination of these factors. Formal aspects of number sense appear weak in children whose mathematics difficulties are linked to impoverished learning environments, but it is unknown if or how this influences the more intuitive aspects of number sense. Does the type of number sense difficulty matter? It may, because intervention needs might differ depending on which components are deficient. As Dr. Berch emphasized earlier in this issue, teachers can make observations to guide adjustments to their instructions for individual children. Knowledge of the importance and complexity of number sense may enhance teachers' observations of this aspect of their students' math performance.

Continued on page 50

Conclusion

To revisit the questions that appear at the beginning of this article, why do some students have difficulty with mathematics? The reasons stem from characteristics of students, teachers, and learning environments discussed in this special issue and many more that were not addressed herein but that are worthy of attention. Additional child characteristics not explored in this issue include (but are not limited to) language (e.g., the language the child speaks, whether the child is monolingual or bilingual, if the child is not fluent in the language of instruction), spatial skills, and the presence of a comorbid disability in another area (e.g., attention or anxiety disorder, etc.), to name a few. Additional influences on the child's learning environment are not limited to the classroom; they include the school and school district, but also the home and the child's larger community. In summary, there are many reasons why children may have difficulty with mathematics.

How often are these difficulties rooted in a mathematical learning disability (MLD)? The quick answer to this question comes from incidence studies, which determine the frequency of a given condition. Incidence studies conducted in the U.S. and in many other countries indicate that approximately 6–10% of school age children have MLD. This finding means that many children with mathematical difficulties may not have MLD. What seems to differentiate MLD—defined here as a cognitively-based difficulty with mathematics—from other forms of mathematical difficulties is a poorly developed number sense.

How early can we identify which children are at risk for poor mathematics outcomes? To date, there is no single universal indicator of this risk, but a poor sense of number in kindergarten is a strong predictor of poor math outcomes later (e.g., Jordan, Kaplan, Locuniak, & Ramineni, 2007; Mazzocco & Thompson, 2005). How can we diminish this risk? Just as low literacy environments put children at higher risk for poor reading achievement, low numeracy environments promote risk for poor mathematics outcomes. Children benefit from systematic enrichment of literacy and numeracy experiences. But that may not be enough for children with MLD, just as mere exposure to print is insufficient for teaching a child with dyslexia to read. What remains to be seen is whether number sense interventions should have an emphasis on specific components of number sense, depending on the child's difficulty in number and other areas, and to what extent number-rich experiences benefit all children with MLD and other forms of MD. These questions are a focus of much ongoing research because of the growing realization of how much number matters in mathematical learning and performance.

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Knowledge and Practice Standards for Teachers of Reading—A New Initiative by the International Dyslexia Association

by Louisa Moats, Ed.D.

As part of the 2010 IDA conference in Phoenix, during the Global Partners Caucus, Dr. Louisa Moats gave a presentation of the International Dyslexia Association's newly published, Knowledge and Practice Standards for Teachers of Reading. *Dr. Moats' remarks were enthusiastically received by the Global Partners in attendance.*

Students with Reading Disabilities Depend on Skilled Teaching

Although dyslexia and related reading and language problems may originate with neurobiological differences, they are mainly treated with skilled teaching. Informed and effective classroom instruction, especially in the early grades, can prevent or at least effectively address and limit the severity of reading and writing problems. Potential reading failure can be recognized as early as preschool and kindergarten, if not sooner. A large body of research evidence shows that with appropriate, intensive instruction, all but the most severe reading disabilities can be ameliorated in the early grades, and students can get on track toward academic success. For those students with persistent dyslexia who need specialized instruction outside of the regular class, competent intervention from a specialist can lessen the impact of the disorder and help the student overcome and manage the most debilitating symptoms.

What is the nature of effective instruction for students at risk? The methods supported by research are those that are explicit, systematic, cumulative, and multisensory, in that they integrate listening, speaking, reading, and writing. The content of effective instruction emphasizes the structure of language, including the speech sound system (phonology), the writing system (orthography), the structure of sentences (syntax), the meaningful parts of words (morphology), meaning relationships among words and their referents (semantics), and the organization of spoken and written discourse. The strategies emphasize planning, organization, attention to task, critical thinking, and self-management. While all such aspects of teaching are essential for students with poor reading and language skills, these strategies also enhance the potential of all students.

Are Teachers Prepared?

Teaching language, reading, and writing effectively, especially to students experiencing difficulty, requires considerable knowledge and skill. Regrettably, the licensing and professional development practices currently endorsed by many states in the U.S. and many countries abroad are insufficient for the preparation and support of teachers charged with preventing and remediating reading problems. Researchers in the U.S. are finding that many teachers are licensed with insufficient knowledge of reading difficulties and literacy instruction (Cunningham, Perry, Stanovich, & Stanovich, 2004; Joshi, Binks, Hougen, Ocker-Dean, Graham, & Smith, 2009; Moats & Foorman, 2003; Spear-Swerling, 2008). Few practitioners are trained in sufficient depth to recognize early signs of risk or to implement research-based instruction (Smartt & Reschly, 2007; Walsh, Glaser, & Wilcox, 2006). In addition, there is as yet no internationally recognized credential for teachers of literacy that is honored throughout the international school community.

To address these gaps and promote more rigorous, meaningful, and effective teacher preparation and professional development, the International Dyslexia Association (IDA) has adopted a comprehensive set of knowledge and practice standards for the training of teachers of reading.

The Purpose of IDA's Standards

IDA's Knowledge and Practice Standards should be used to guide the preparation, certification, and professional development of those who teach reading and related literacy skills in classroom, remedial, and clinical settings. The standards aim to specify what any individual responsible for teaching reading should know and be able to do so that reading difficulties, including dyslexia, may be prevented, alleviated, or remediated. In addition, the standards seek to differentiate classroom teachers from therapists or specialists who are qualified to work with the most challenging students.

Although programs that certify or support teachers, clinicians, or specialists differ in their preparation methodologies, teaching approaches, and organizational purposes, IDA hopes to bring these groups together under a common set of professional standards that will benefit all students with reading and writing difficulties. If a training entity aligns with these standards, the public should be assured that certified individuals are prepared to implement scientifically based and clinically proven best practices.

The standards outline three critical dimensions of teacher preparation: 1) content knowledge necessary to teach reading and writing to students with dyslexia or related disorders or

Continued on page 52

who are at risk for reading difficulty; 2) practices of effective instruction; and 3) ethical conduct expected of professional educators and clinicians. Regular classroom teachers should also have the foundational knowledge of language, literacy development, and individual differences because they share responsibility for preventing and ameliorating reading problems.

The standards may be used for several purposes, including but not limited to

- course design within teacher certification programs;
- practicum requirements within certification programs;
- criteria for endorsement of a teacher training program by IDA;
- criteria for identifying qualified professionals who are eligible to receive referrals through IDA offices; and
- a content framework for the development of licensing or certification examinations.

IDA's Strategic Goals

The standards and practices work of IDA will be a long-term endeavor. With the leadership of the National Board and headquarters staff, we will begin in 2011 to endorse programs that accredit and certify teachers in accordance with the IDA standards. Simultaneously, we will establish criteria and a review process for accreditation of international training programs, and develop processes whereby individual teachers who are unable to attend an endorsed or accredited program can be certified directly through IDA. All this will require time and collaboration among our office staff, National Board, and global partners, but IDA is prepared to play a central role in assuring that teachers are qualified for this rewarding profession.

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IDA's Knowledge and Practice Standards can be downloaded at www.interdys.org

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